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Effects of the vertical CoM motion on energy consumption for walking humanoids

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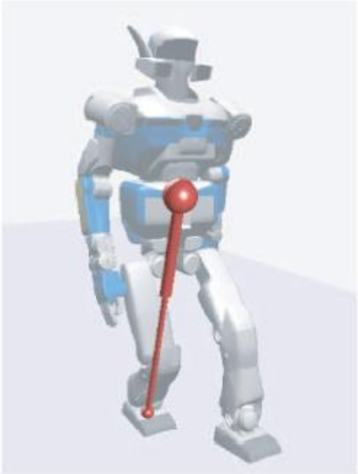
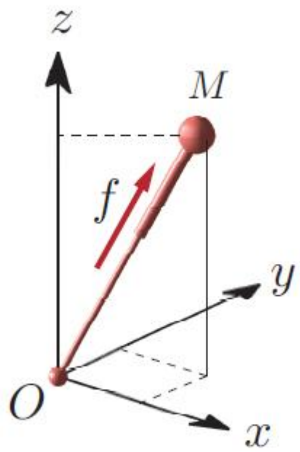
Introduction

- The robot is represented by the inverted pendulum model.
- Motivation : release the vertical motion of pendulum CoM from being at a constant height.
- We will consider two cases :





[Kajita 2004]

The linear inverted pendulum	The general inverted pendulum
$z = cnst$	$z \neq cnst$
Analytical solution	Numerical solution
fast and easy calculations	

What are the differences between the two models ?

Plan of the presentation

- Inverted pendulum dynamics
- Modeling
  - Walking cycle
  - The studied robot
  - Dynamic model
- Sthenic criterion
- Results
- Conclusion & Futur work

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Inverted pendulum dynamics

- The dynamics of a humanoid robot can be approximated by the inverted pendulum model.

$$\begin{cases} x_p = x - \dfrac{z - z_p}{\ddot{z} + g}\ddot{x} \\ \\ y_p = y - \dfrac{z - z_p}{\ddot{z} + g}\ddot{y} \end{cases}$$

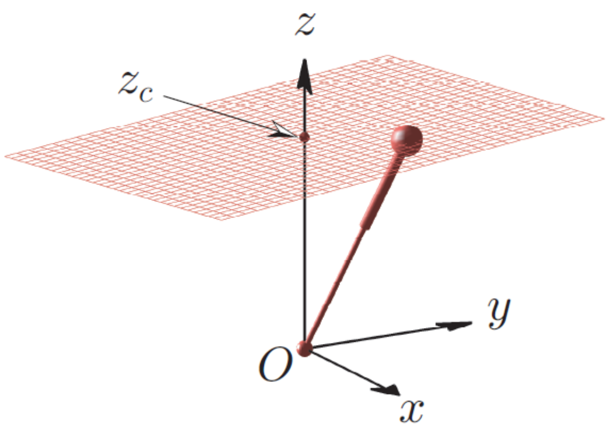
$P = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix}$ are the ZMP coordinates.

$C = \begin{bmatrix} x \\ y \\ \text{z} \end{bmatrix}$ are the CoM coordinates.

g is the gravity acceleration.

Linear inverted pendulum

Constant height of CoM



[Kajita 2004]

$$z = cnst = z_c$$

$$\ddot{z} = 0$$

$$\begin{cases} x_p = x - \dfrac{z - z_p}{\ddot{z} + g}\ddot{x} \\ y_p = y - \dfrac{z - z_p}{\ddot{z} + g}\ddot{y} \end{cases} \Rightarrow \begin{cases} x_p = x - \dfrac{z_c - z_p}{g}\ddot{x} \\ y_p = y - \dfrac{z_c - z_p}{g}\ddot{y} \end{cases}$$

Linear differential system

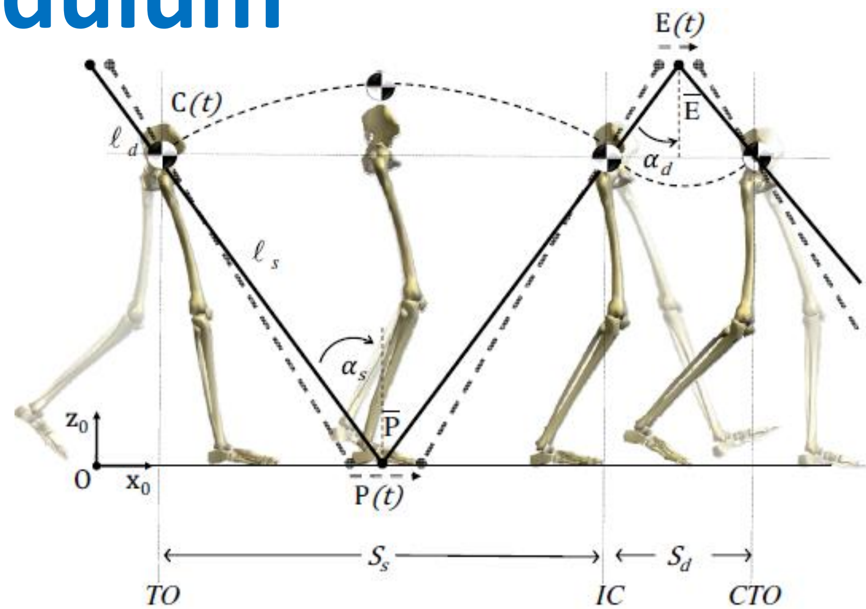
Analytical solution : $x = x(0)\cosh(t/T_c) + T_c\ \dot{x}(0)\sinh(t/T_c)$

Where : $T_c = \sqrt{\frac{z_c}{g}}$

General inverted pendulum

$z \neq cnst$

$$\begin{cases} x_p = x - \dfrac{z - z_p}{\ddot{z} + g}\ddot{x} \\ y_p = y - \dfrac{z - z_p}{\ddot{z} + g}\ddot{y} \end{cases}$$



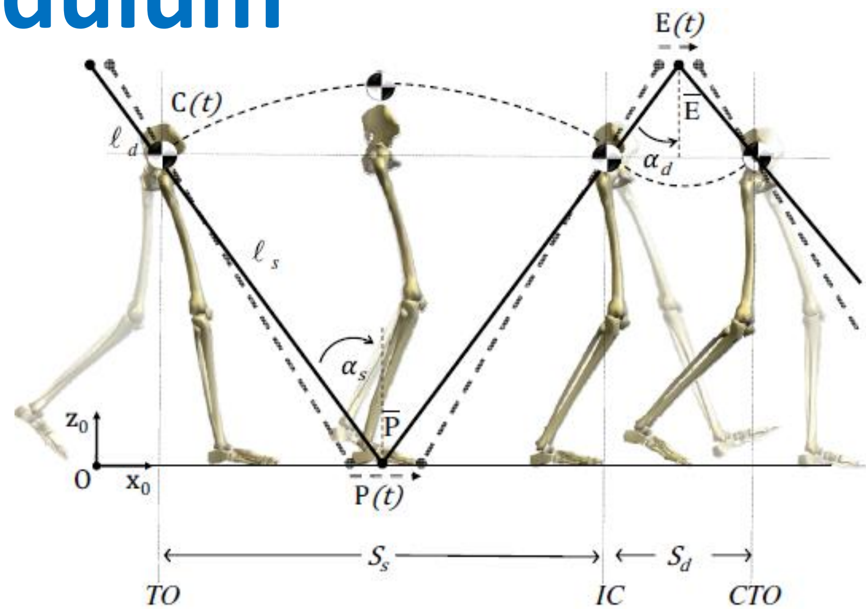
[Hayot et al 2013]

Z is defined according to the study of Hayot et al about the CoM trajectory during normal gait using a multi-body model of human subjects.

General inverted pendulum

$z \neq cnst$

$$\begin{cases} x_p = x - \dfrac{z - z_p}{\ddot{z} + g}\ddot{x} \\ y_p = y - \dfrac{z - z_p}{\ddot{z} + g}\ddot{y} \end{cases}$$



[Hayot et al 2013]

From this study, we notice:

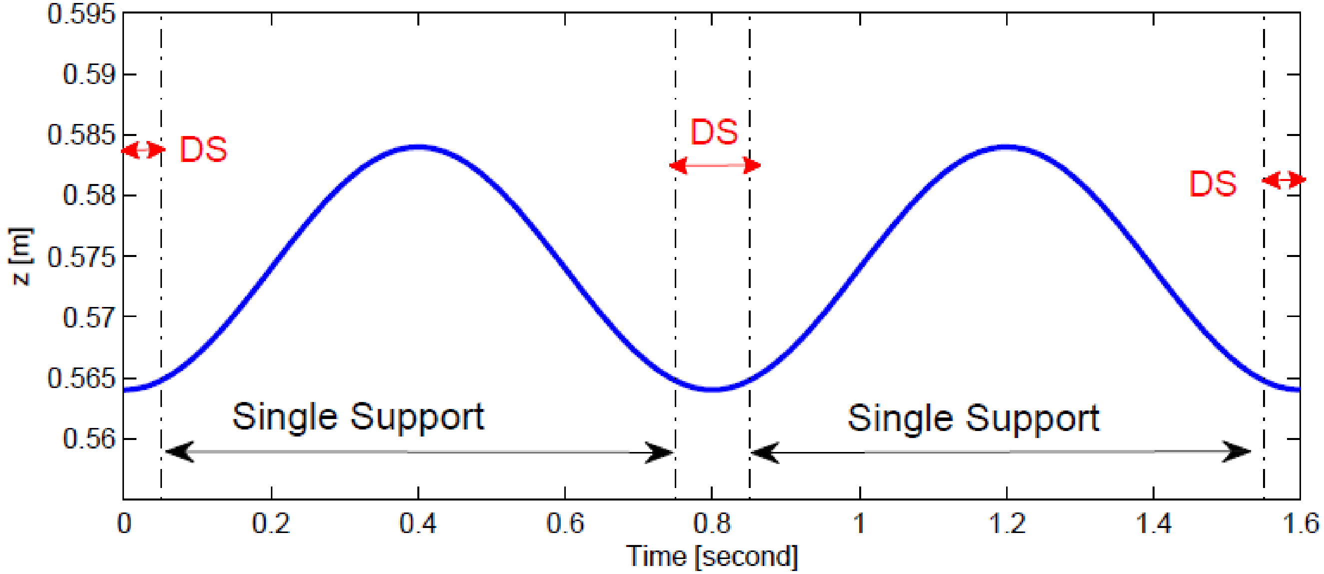
- The vertical position of CoM oscillates along the average value of the CoM height.
- It reaches its maximal value at midstance.
- It reaches its minimal value at the middle of the double support phase.

According to this description, we take a sinusoidal function of time:

$$z = z_c + A\ \cos(\omega t + \emptyset)$$

Vertical displacement of CoM

$$z = z_c + A\ \cos(\omega t + \emptyset)$$



A is the vertical amplitude of the CoM altitude.

T being the period of one step. $\omega = 2\pi/T$

Φ is an angle depending on the first phase of the motion (single support SS or double support DS).

Differential equations system

By replacing z and $\ddot{z}$ in the differential equations system, we have:

$$z = z_c + A\ \cos(\omega t + \emptyset)$$

$$x_p = x - \frac{z - z_p}{\ddot{z} + g}\ddot{x} \quad \Longrightarrow \quad \ddot{x} = \frac{g - \omega^2 A\cos(\omega t + \emptyset)}{z_c + A\cos(\omega t + \emptyset)}\,(x - x_p)$$

The second order differential equation is non-linear and non-homogeneous.

Its solving is performed numerically.

The same for the motion in y direction.

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Walking cycle

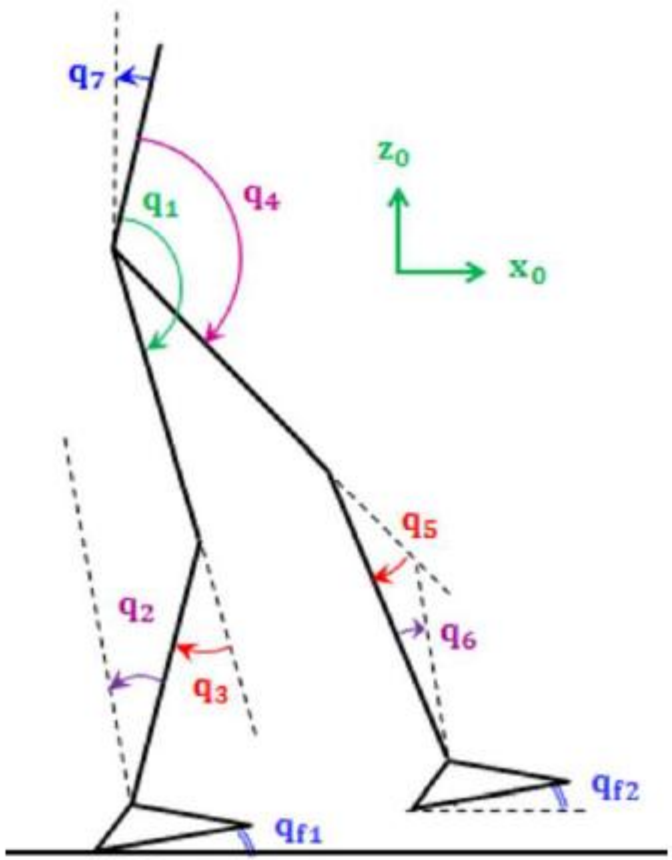
- The walking cycle is defined by two successive steps (right and left legs).
- One step is composed of a single support phase (SS) on the stance leg, and a double support phase (DS).
- The system parameters for positions, velocities and accelerations are equal at the beginning and at the end of each cycle.

Hypotheses

- Feet soles remain parallel to the ground.
- The trunk segment remains vertical.
- CoM and waist segment have the same velocity profiles.
- Feet velocity and acceleration are equal to zero at foot strike.

Studied robot

- We consider a 2D humanoid robot.
- It is composed of 6 actuators to control its body movements in the sagittal plane.
  (2 ankles, 2 knees and 2 hips)



- The generalized coordinates vector is given :

$$q = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6 \quad q_7 \quad x \quad z]^t$$

$q_1, \dots, q_6$ are the joint variables

q_7 is the orientation of the trunk

x and z are the Cartesian coordinates of the hip.

Dynamic model

$$\begin{cases} D\ddot{q} + C\dot{q} + G = B\Gamma + {J_i}^t R_i & \text{In single support on leg i} \\ D\ddot{q} + C\dot{q} + G = B\Gamma + {J_1}^t R_1 + {J_2}^t R_2 & \text{In double support phase} \end{cases}$$

D is the inertia matrix.

C is the vector of Coriolis forces.

G is the gravity forces.

B is the actuation matrix.

$R_1 = \begin{bmatrix} R_{1x} \\ R_{1z} \\ M_{1y} \end{bmatrix}$ & $R_2 = \begin{bmatrix} R_{2x} \\ R_{2z} \\ M_{2y} \end{bmatrix}$ are the ground reaction forces

Dynamic model

<u>In single support phases :</u> $D\ddot{q} + C\dot{q} + G = B\Gamma + {J_i}^t\, R_i$

We have 9 unknowns in $\Gamma\ (6)$ and $R_i(3)$

So the 9 equations are sufficient.

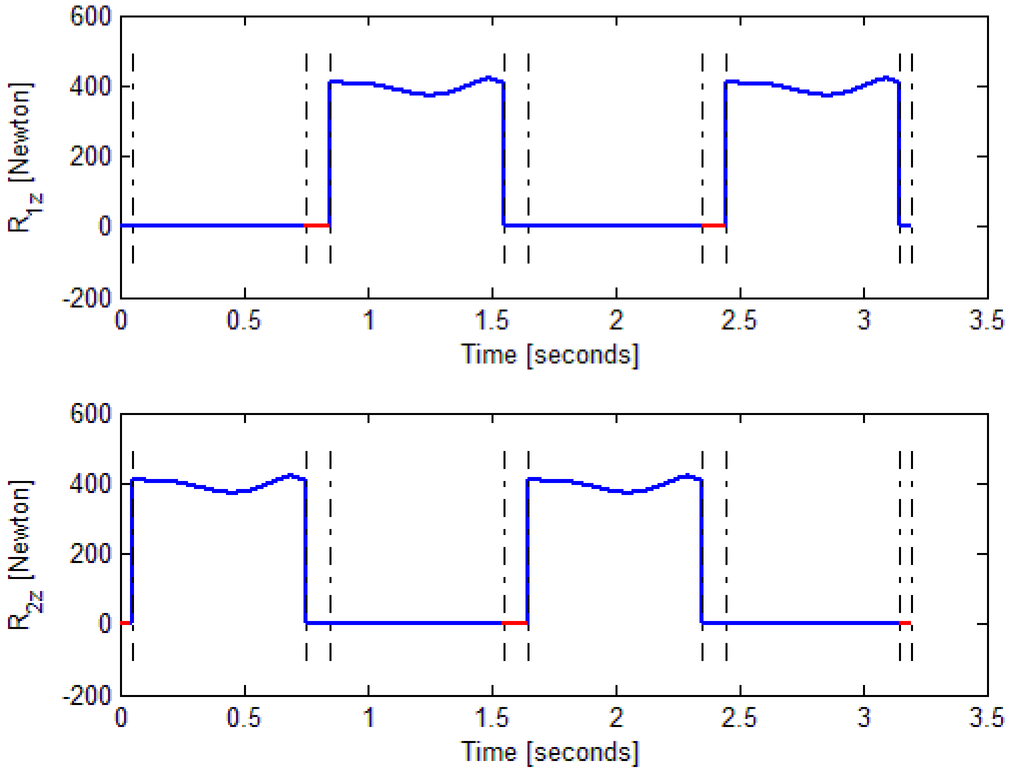
<u>In double support phases :</u> $D\ddot{q} + C\dot{q} + G = B\Gamma + {J_1}^t\, R_1 + {J_2}^t\, R_2$

We have 12 unknowns in $\Gamma\ (6)$ and $R_1(3)$ and $R_2(3)$

So the 9 equations are not sufficient !

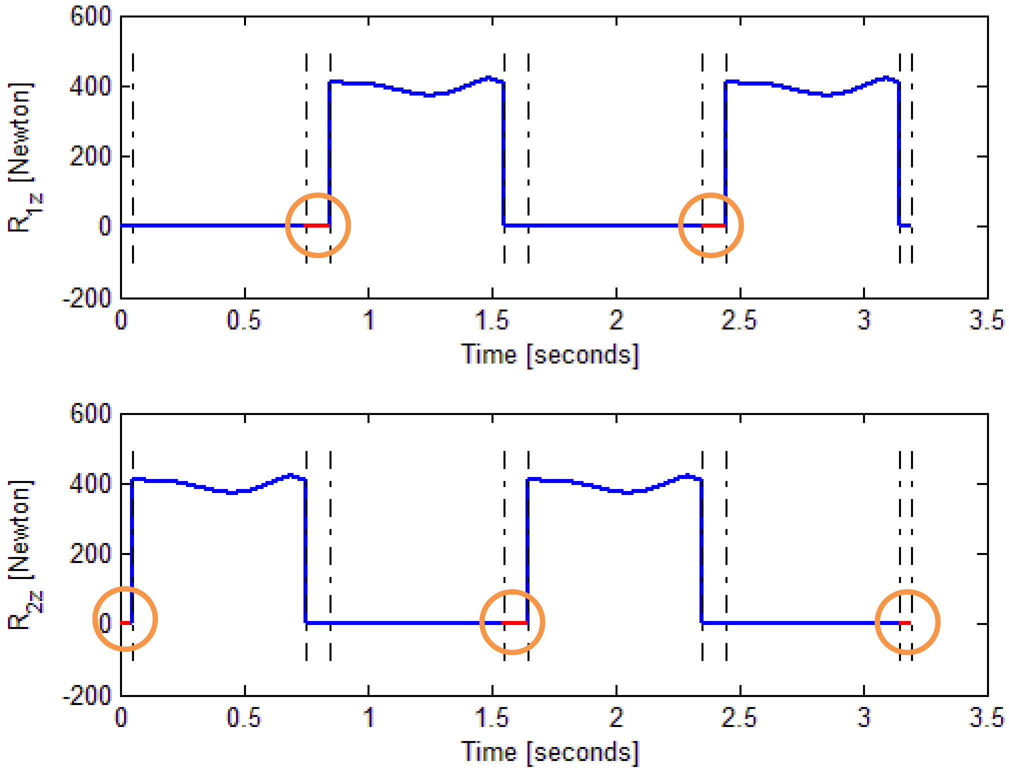
Dynamic model

We solve the problem in the SS phases as the number of unknowns = the number of equations :
We show the result for the vertical component, it is the same for the horizontal force and the moment.



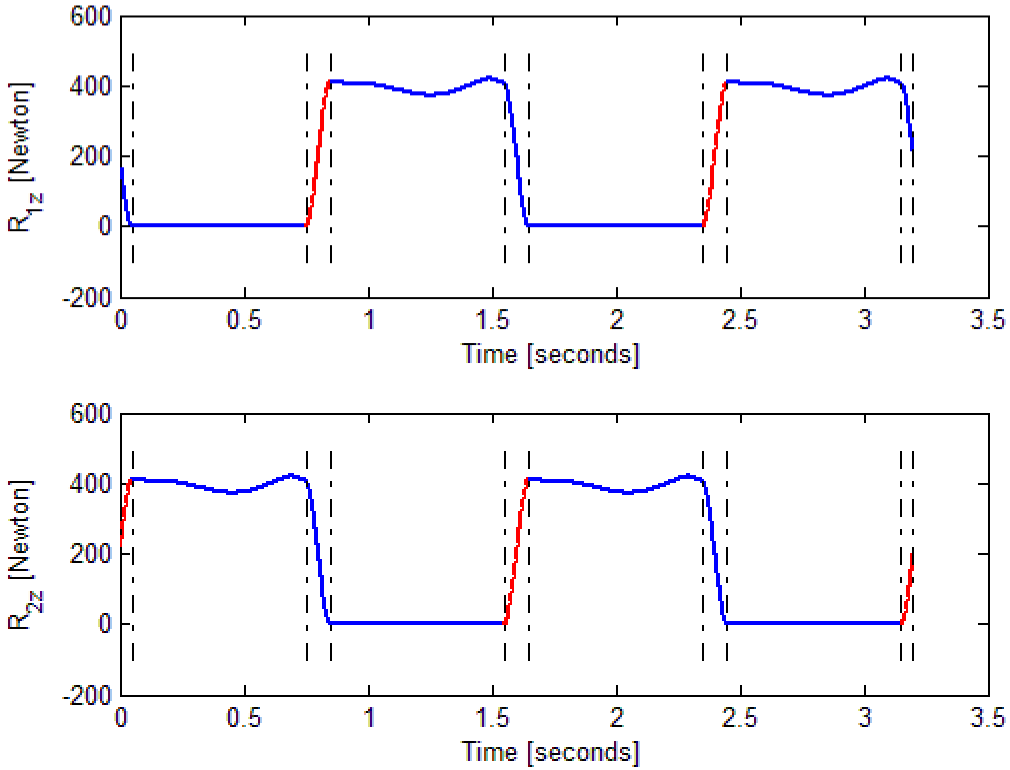
Dynamic model

- In DS phases, the number of unknowns > the number of equations !
- We choose to fix the 3 components of ground reaction forces exerted on the foot that was swinging before the considered double support.



Dynamic model

Then the 3 components of the chosen reaction force can be defined by a third order polynomial meeting the boundary conditions: continuity of the force and continuity of the force derivative.



Dynamic model

For example, In DS phases after the swinging of foot 2, we specify R_2 as a polynomial function, then R_1 and joint torques can be calculated by :

$$\begin{bmatrix} \Gamma \\ R_1 \end{bmatrix} = \begin{bmatrix} B & {J_1}^t \end{bmatrix}^{-1} \left[D\ddot{q} + C\dot{q} + G - {J_2}^t R_2 \right]$$

By repeating this procedure for the horizontal force and the moment, we fix 3 components of reaction forces, so we can solve the dynamic model to obtain joint torques and the second ground reaction vector.

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Sthenic criterion

The sthenic criterion is defined as the quadratic actuated torque per unit of distance

$$E = \frac{1}{d} \int_{t_0}^{t_f} \Gamma^t \, \Gamma \, \mathrm{dt}$$

where t_0 and t_f denote the beginning and ending instants of the total observed motion, d is the traveled distance.

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Results

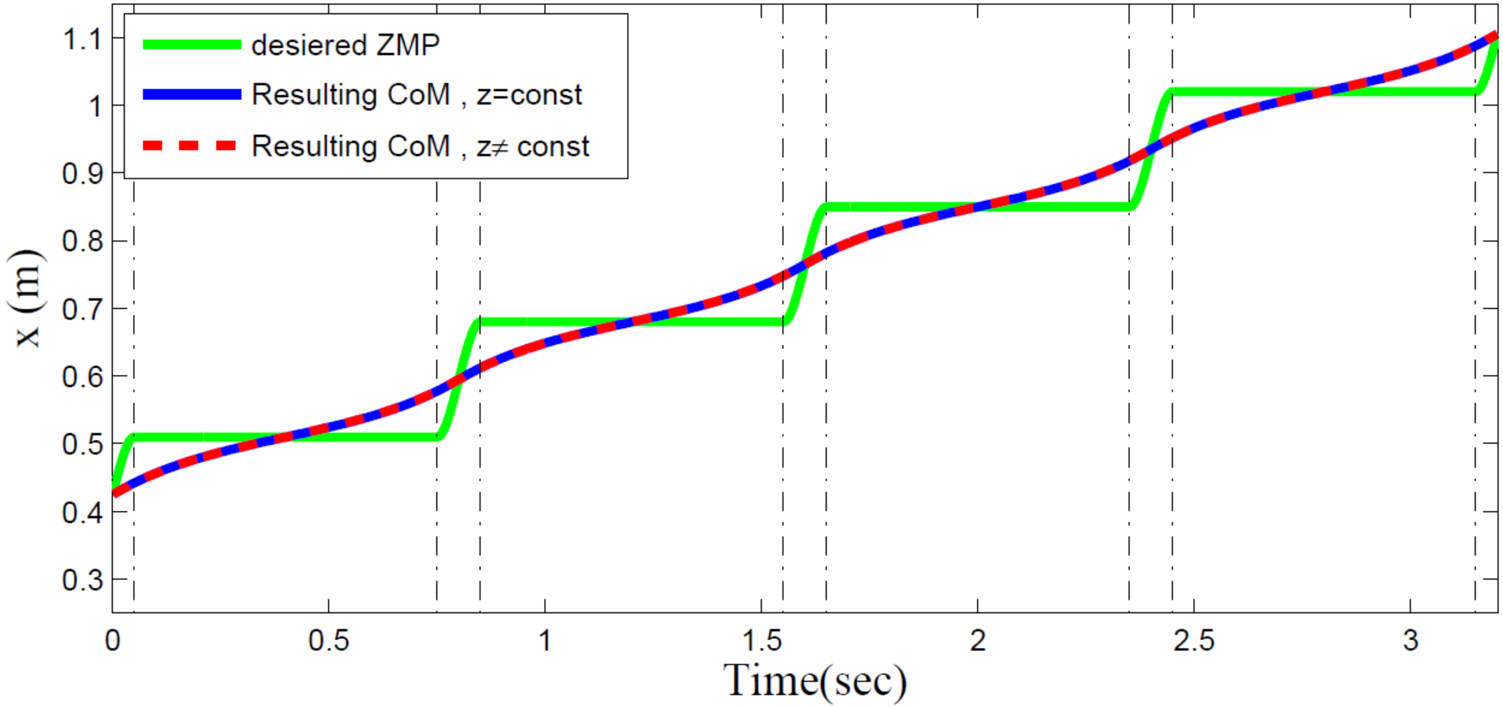
The step length was set to 0.348 [m] and the cycle duration to 1.6 [s], with 0.1 [s] for each double support and 0.7 [s] for each single support phases.

We will compare the results in two cases :

- Walking gait with a constant height of CoM (linear inverted pendulum).
- Walking gait with a sinusoidal height of the CoM of amplitude 2A =2[cm].
- The only difference between the two walking gaits is the vertical displacement of the CoM.

Results

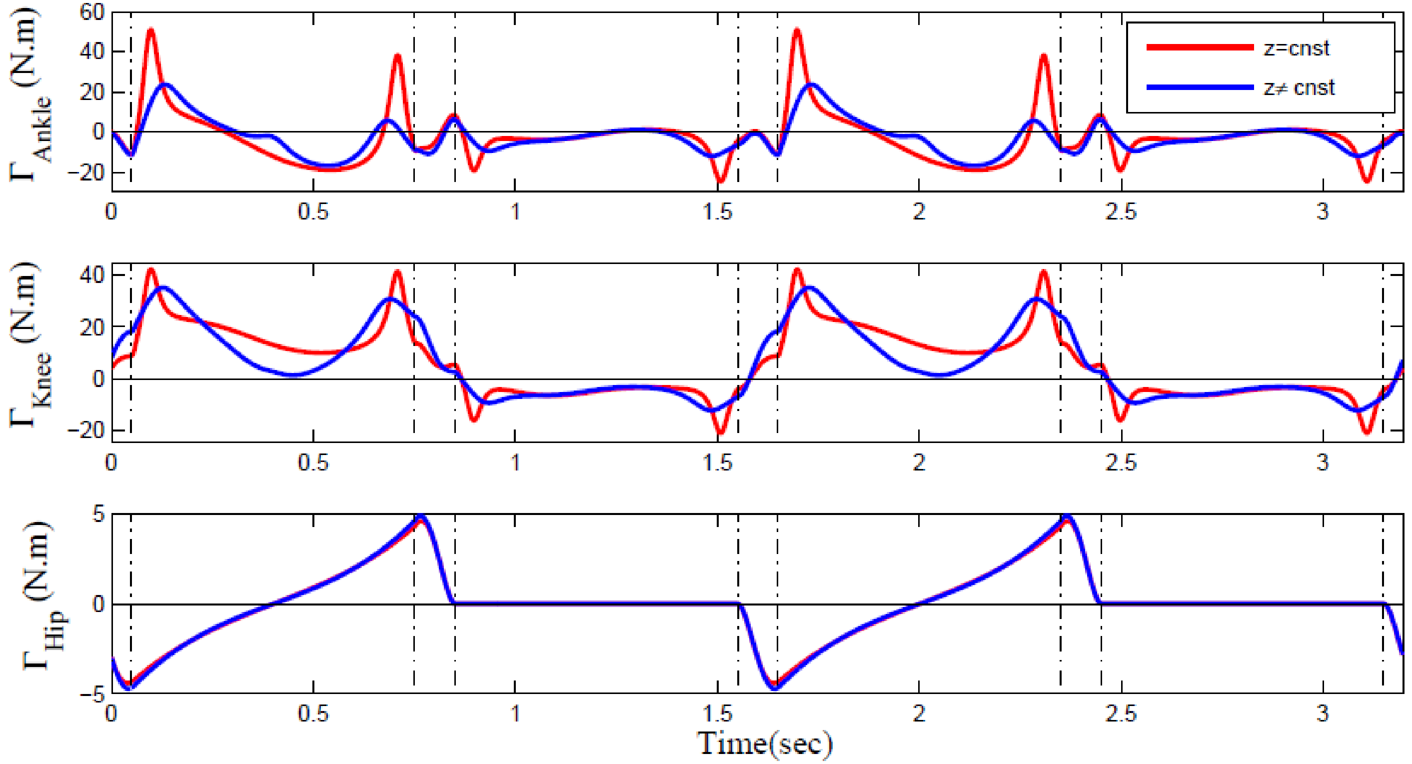
Horizontal advancement of CoM :



- We notice that the two forward CoM trajectories are almost identical

Results

Joint torques

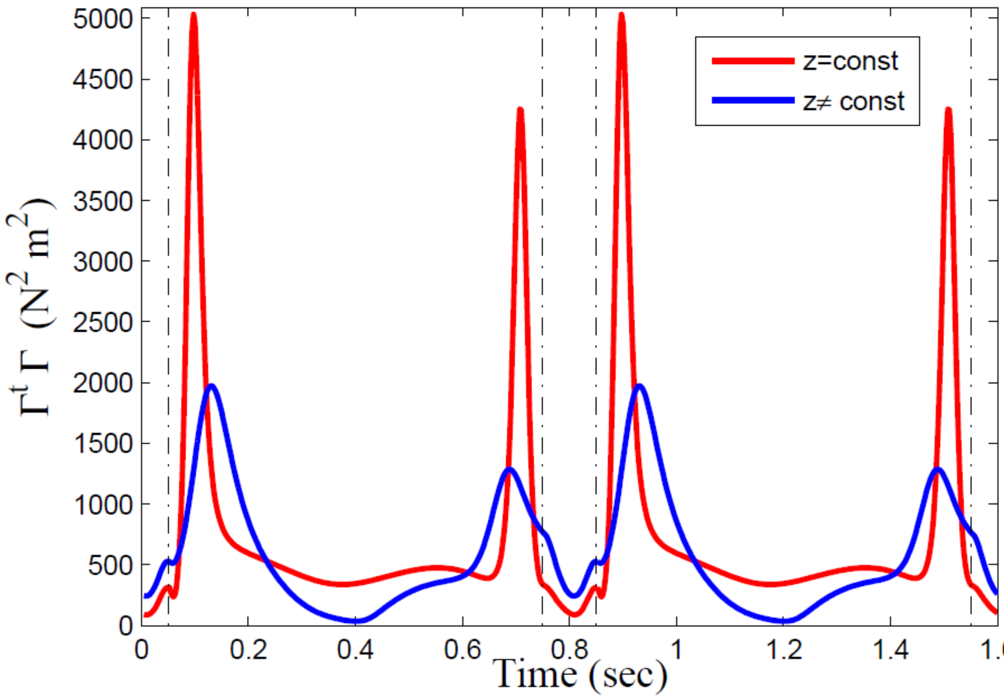


For the knees and ankles joints, They are smoother when the CoM of the robot oscillates sinusoidally in the vertical direction. Also these torques show much lower values, particularly at the boundaries between single and double support phases.

Results

Quadratic actuated torque

$$E_0 = \Gamma^t \Gamma$$



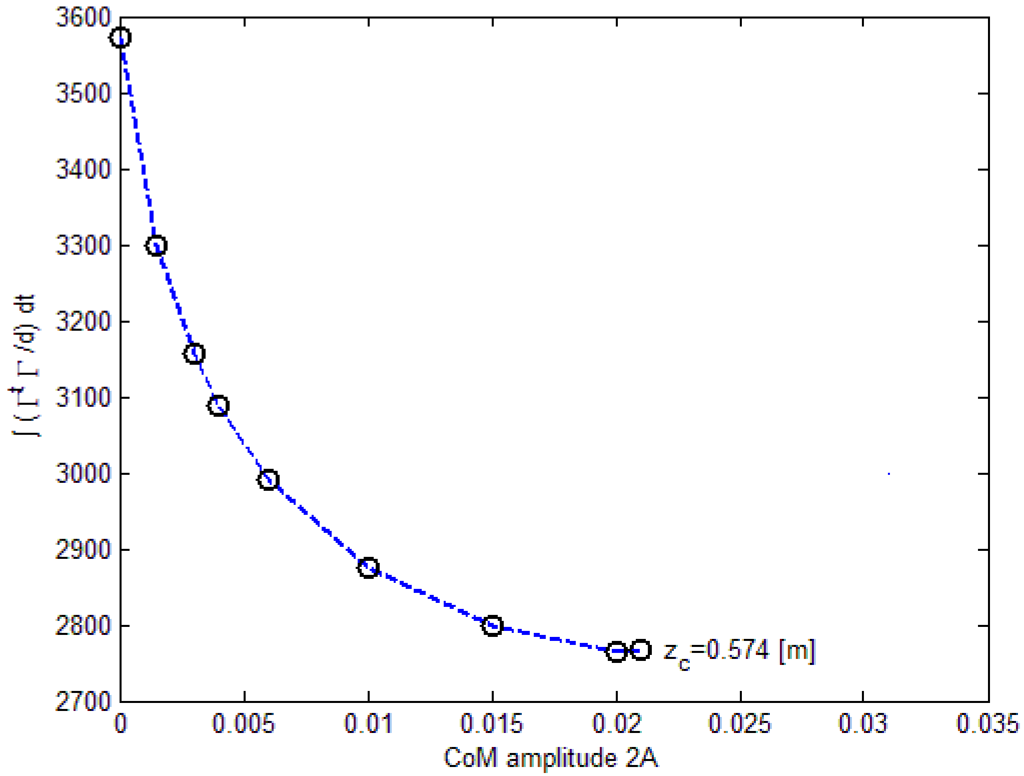
(b)

- E_0 is very high at both ends of the single support phases, for the two cases z = cnst and 2A = 2 [cm].

- E_0 is much lesser with 2A = 2 [cm] than with z = cnst.

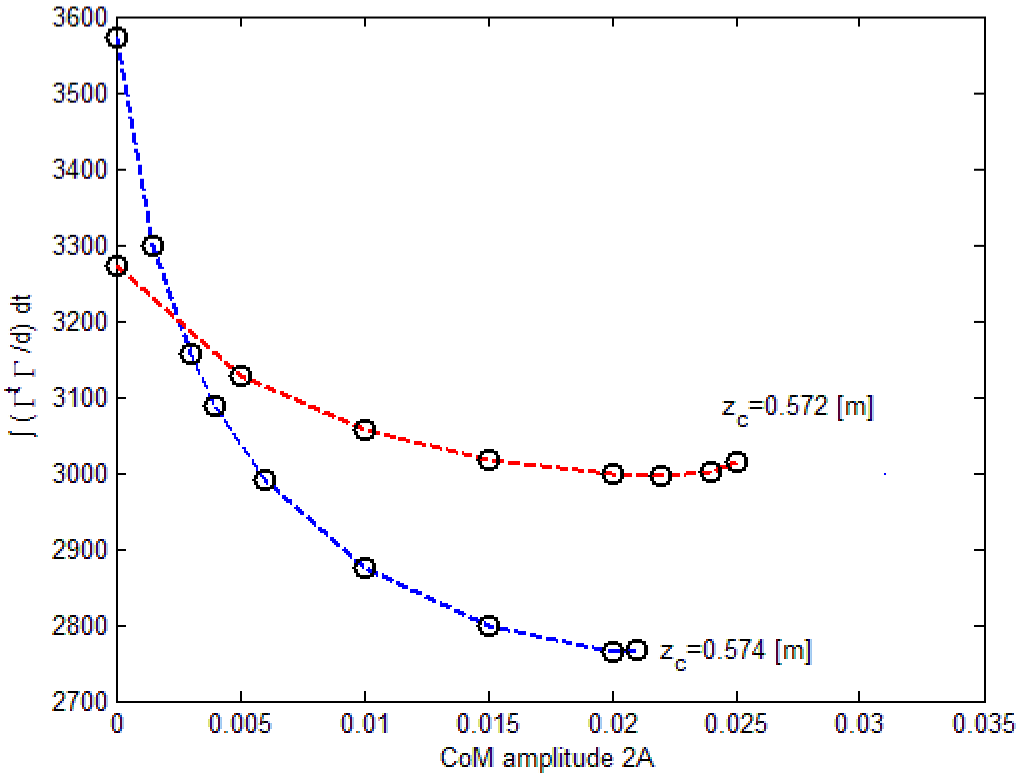
Results

Sthenic criterion



Results

Sthenic criterion



We notice that the sthenic criterion decreases considerably when 2A increases in the range [0,2] [cm]

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Conclusion

- We proposed an analysis of the effect of CoM vertical magnitude on the energy consumption for humanoid walking gait.
- We have compared two cases in a 2D simulation : the classical LIP model with constant height of CoM and general IP model with oscillating CoM height.
- The comparison allowed to conclude that :

1. For both IP models, the highest actuator torques occur at the change of single and double support phases.
2. The use of a variable CoM height considerably reduces the torque solicitations at the change of support, and at midstance.

Future work

- Find the optimal value of the CoM vertical magnitude according to step length and walking velocity.
- Expand the study of the CoM vertical magnitude to 3D walking,

Thank you for your attention.

Questions ?