Robust control of mechanical systems under unilateral constraints with application to a biped robot

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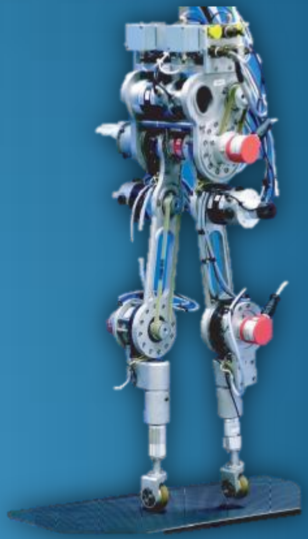

Outline

- Introduction
- Bipedal walking as a hybrid system
- Robust control synthesis
- Numerical results
- Future work

Introduction

Hybrid systems

Hybrid Systems

Autonomous-switching hybrid system

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Continuous-controlled autonomous-switching hybrid

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Autonomous-impulse hybrid system

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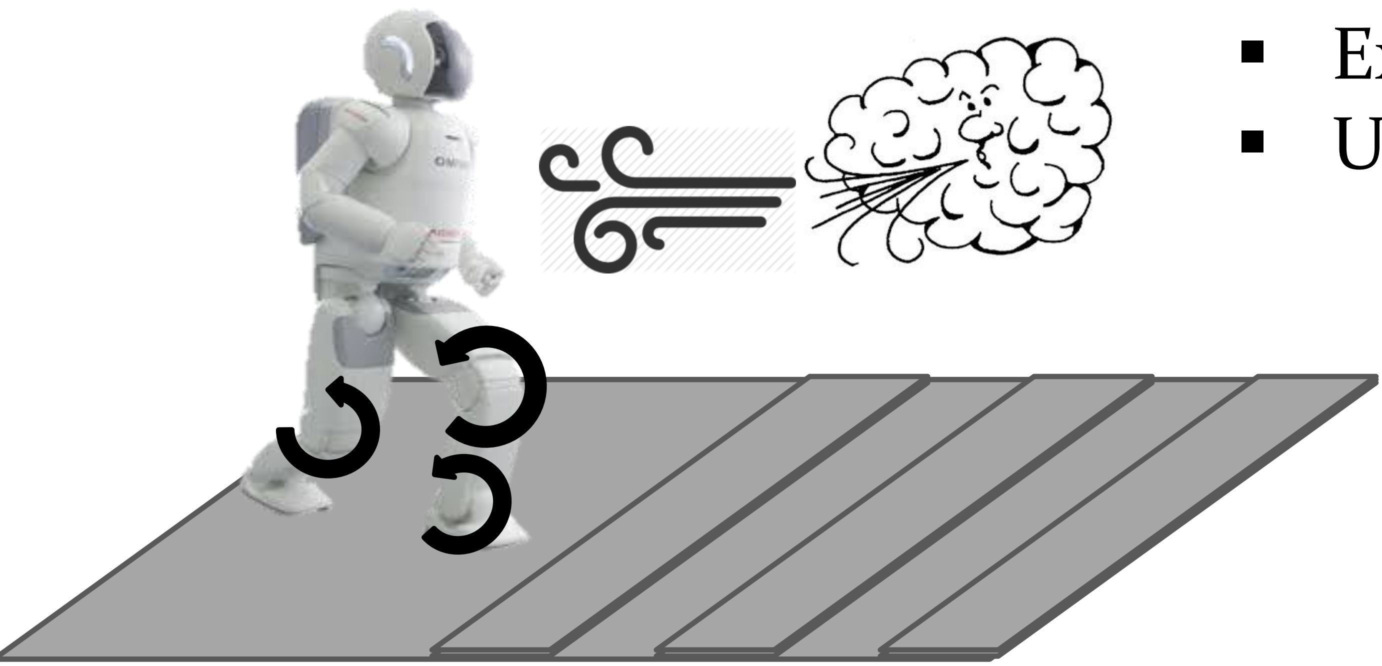
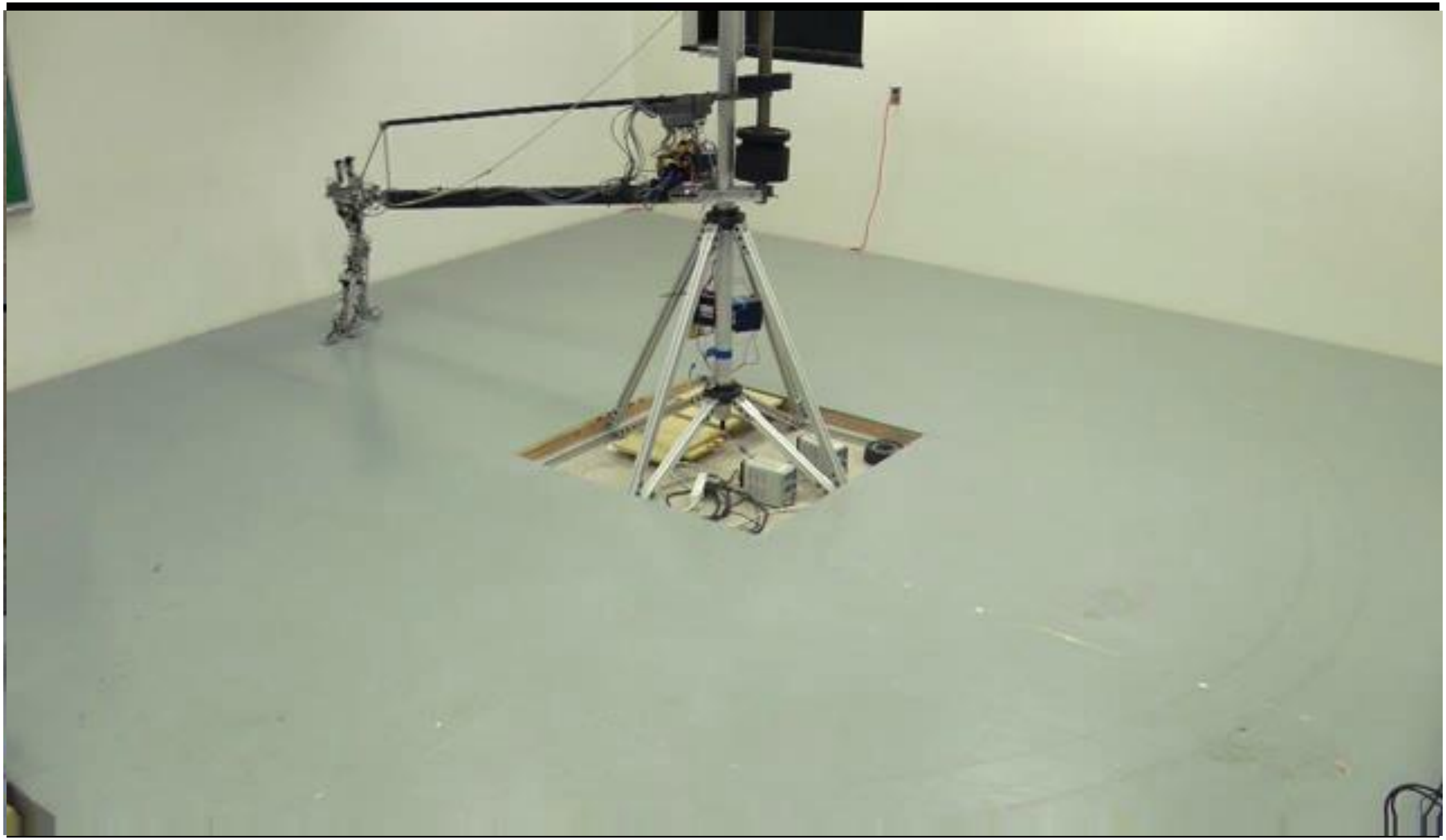
Systems with unilateral constriants (Brogliato, 1996)

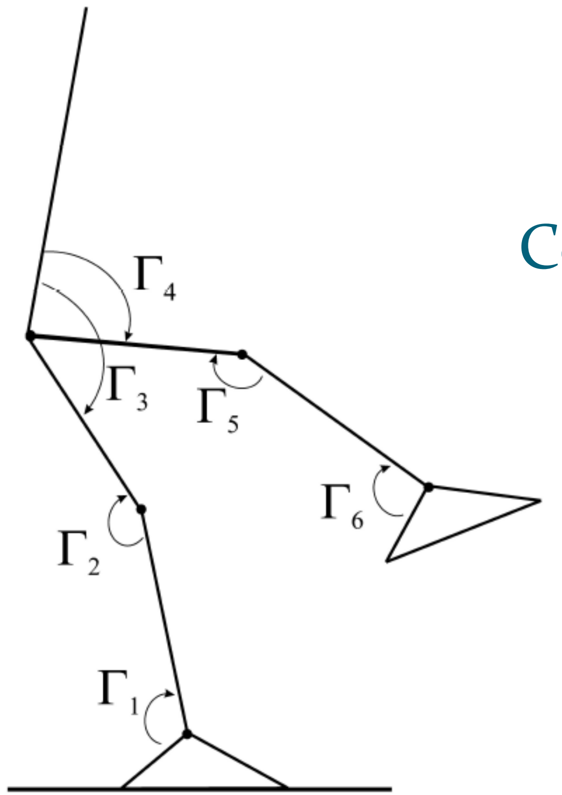
Disturbances



- Friction
- External forces
- Uneven ground



Disturbance attenuation



Compensator

Disturbance attenuator

Actuators

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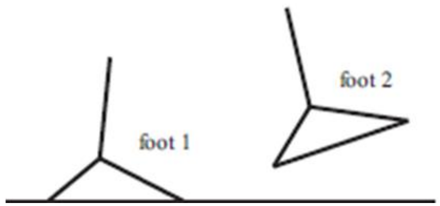
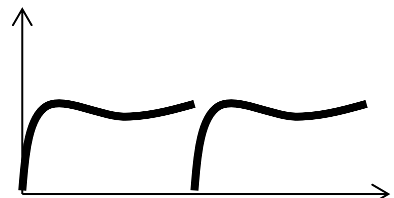
Disturbance attenuation



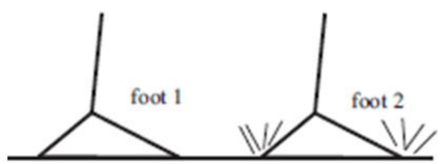
Continuous disturbance attenuator



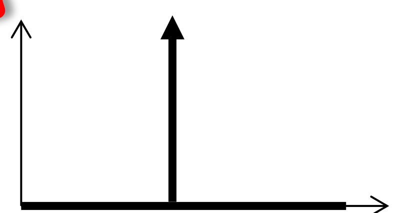
$$0 = x^{\mathrm{T}}(A_{\mathrm{c}}^{\mathrm{T}} P + P A_{\mathrm{c}} + R_{1\mathrm{c}} + \gamma_{\mathrm{c}}^{-2} P D_{\mathrm{c}} D_{\mathrm{c}}^{\mathrm{T}} P - P S_{\mathrm{c}} P)x$$

$$u_{\mathrm{c}} = \phi_{\mathrm{c}}(x) = -R_{2\mathrm{c}}^{-1} B_{\mathrm{c}}^{\mathrm{T}} P x$$

Trés difficile!!



Discrete disturbance attenuator



$$0 < \gamma_{\mathrm{d}}^2 I_{d_{\mathrm{d}}} - D_{\mathrm{d}}^{\mathrm{T}} P D_{\mathrm{d}}$$

$$0 = x^{\mathrm{T}}(A_{\mathrm{d}}^{\mathrm{T}} P A_{\mathrm{d}} - P + R_{1\mathrm{d}} + A_{\mathrm{d}}^{\mathrm{T}} P D_{\mathrm{d}}(\gamma_{\mathrm{d}}^2 I_{d_{\mathrm{d}}} - D_{\mathrm{d}}^{\mathrm{T}} P D_{\mathrm{d}})^{-1} D_{\mathrm{d}}^{\mathrm{T}} P A_{\mathrm{d}} - P_a^{\mathrm{T}} R_{2a\mathrm{d}}^{-1} P_a)x$$

$$u_{\mathrm{d}} = \phi_{\mathrm{d}}(x) = -R_{2a\mathrm{d}}^{-1} P_a x$$

Impulsive force!

Haddad et al. (2005)

General Objective

- Address the problem of <u>disturbance attenuation for mechanical systems under unilateral constraints</u>

- Consider bounded exogeneous <u>disturbances on</u>
  - <u>position measurements</u>
  - <u>continuous phase</u>
  - <u>impact phase</u>.

- Avoid the use of impulsive inputs

- Consider that we may only have position measurments

Bipedal walking as a hybrid system

Problem statement

- Consider the nonlinear mechanical system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \Phi(x_1, x_2, t) + \Psi_1(x_1, x_2, t)w + \Psi_2(x_1, x_2, t)u \quad (1)$$

$$z = h_1(x_1, x_2, t) + k_{12}(x_1, x_2, t)u \quad (2)$$

Continuous dynamics
( )

$$y = h_2(x_1, x_2, t) + k_{21}(x_1, x_2, t)w \quad (3)$$

Measurements

$$x_1(t_i^+) = x_1(t_i^-)$$
$$x_2(t_i^+) = \mu_0(x_1(t_i), x_2(t_i^-), t_i) + \omega(x_1(t_i), x_2(t_i^-), t_i)w_i^d \quad (4)$$

$$z_i^d = x_2(t_i^+) \quad (5)$$

Discrete dynamics
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-attenuator

Asymptotical Stability

Disturbance Attenuation

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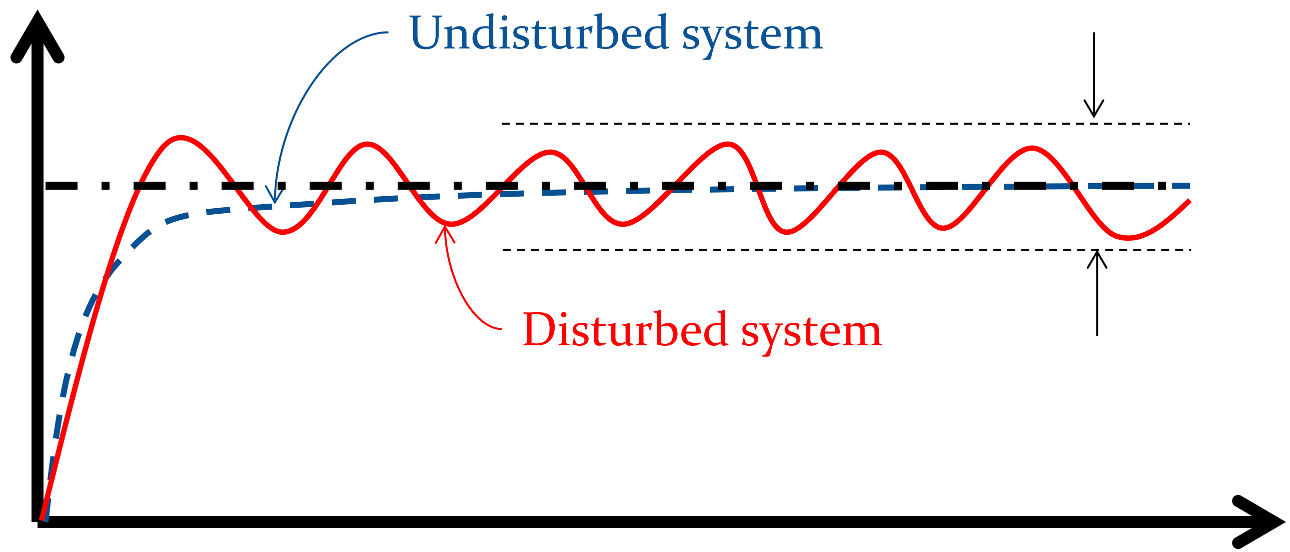
control

Disturbance attenuation with



Local space-state solution

- The subsequent local analysis involves the linear - control problem for the system within the impact free intervals ( )

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B_1}(t)\mathbf{w} + \mathbf{B_2}(t)\mathbf{u}, \quad (20)$$

$$\mathbf{z} = \mathbf{C_1}(t)\mathbf{x} + \mathbf{D_{12}}(t)\mathbf{u}, \quad (21)$$

$$\mathbf{y} = \mathbf{C_2}(t)\mathbf{x} + \mathbf{D_{21}}(t)\mathbf{w}, \quad (22)$$

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Local space-state solution

The output feedback consist of:

Robust State Observer

$$(\quad) \left[- \quad (\quad) \quad (\quad) \qquad (\quad) \quad (\quad) \right] \; (\,) \qquad (\,) \; [\qquad\qquad]$$
$$(\quad)$$

Feedback law

$$(\quad) \qquad (\,)$$

Gain Matrices obtained from optimization !!

And it is a local solution of the -control problem for the nonlinear system (1)-(5)

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Local space-state solution

( ) *and* ( ) *are bounded, symmetrical, positive definite solutions of the differential* *Riccati* *equation system*

$$-\dot{\mathbf{P}}_\varepsilon(t) = \mathbf{P}_\varepsilon(t)\mathbf{A}(t) + \mathbf{A}^\top(t)\mathbf{P}_\varepsilon(t) + \mathbf{C_1}^\top(t)\mathbf{C_1}(t)$$

$$+\mathbf{P}_\varepsilon(t)[\frac{1}{\gamma^2}\mathbf{B}_1\mathbf{B_1}^\top - \mathbf{B}_2\mathbf{B_2}^\top](t)\mathbf{P}_\varepsilon(t) + \varepsilon\mathbf{I}$$

They are only dependent of the trajectory, so if it is known, and can be calculated *a priori*

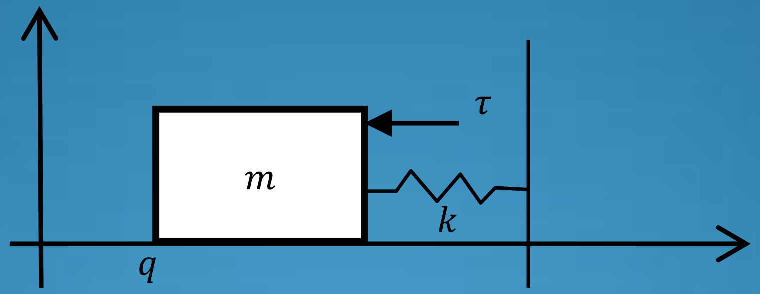
$$\dot{\mathbf{Z}}_\varepsilon(t) = \mathbf{A}_\varepsilon(t)\mathbf{Z}_\varepsilon(t) + \mathbf{Z}_\varepsilon(t)\mathbf{A}_\varepsilon^\top(t) + \mathbf{B_1}(t)\mathbf{B_1}^\top(t)$$

$$+\mathbf{Z}_\varepsilon(t)[\frac{1}{\gamma^2}\mathbf{P}_\varepsilon\mathbf{B}_2\mathbf{B_2}^\top\mathbf{P}_\varepsilon - \mathbf{C_2}^\top\mathbf{C_2}](t)\mathbf{Z}_\varepsilon(t) + \varepsilon\mathbf{I}$$

$$with\ \mathbf{A}_\varepsilon(t) = \mathbf{A}(t) + \tfrac{1}{\gamma^2}\mathbf{B_1}(t)\mathbf{B_1}^\top(t)\mathbf{P}_\varepsilon(t)$$

Example

Tracking of a Mass-spring-damper-barrier system



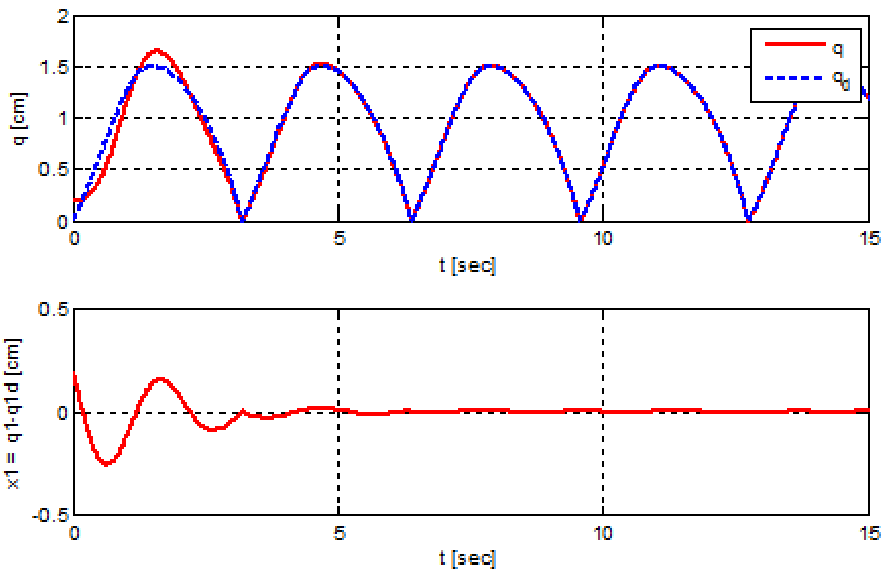
Simulation results

- The simulation shown was performed using Matlab and the parameters from the table:

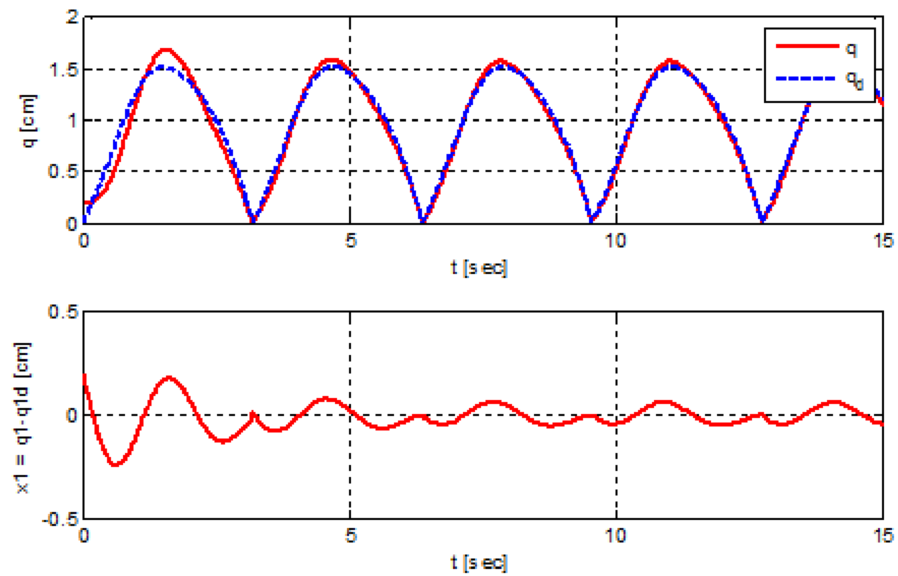
Param	Value	Param	Value
k	$10\ N/m$	ρ_v	1
b	$1\ N/m/s$	ε	0.01
m	$1\ kg$	w_i^d	$0.2q_2\ m/s$
e	0.5	w_1	$0.1q_2 + 0.1\mathrm{sign}(q_2)\ N$
ρ_p	1	w_2	$0.1\sin(1.5t)\ m$
$q(t_o)$	$0.2\ m$	$\dot{q}(t_0)$	$0.8\ m/s$
$\xi_1(t_o)$	$0\ m$	$\xi_2(t_0)$	$0.8\ m/s$

Simulation results

Undisturbed System

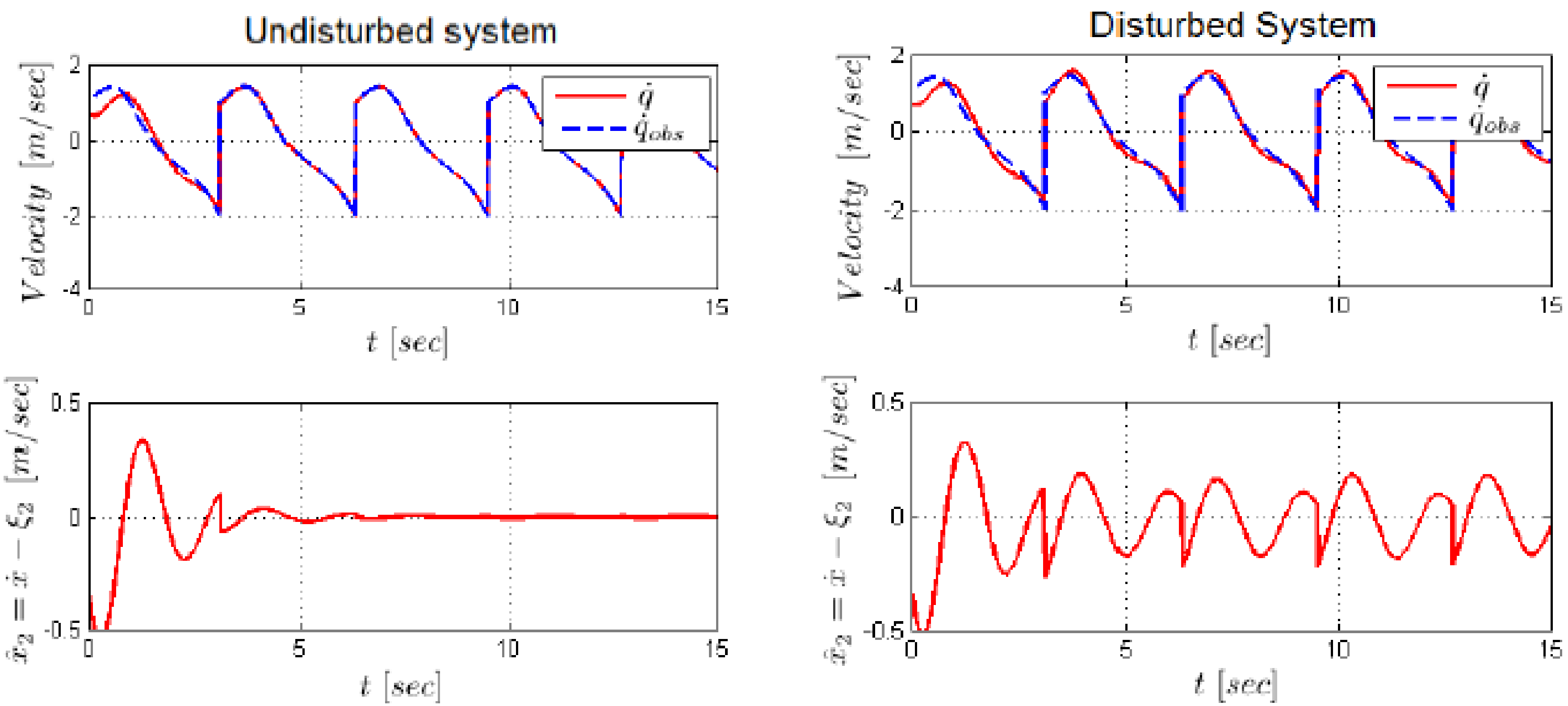


Disturbed System



The system tracks the desired trajectory in a sound manner despite the disturbances affecting both the free-motion (coulomb friction) and transition phases (deviation from restitution coefficient), as well as disturbances in the position measurement, while asymptotically stabilizing the error for the undisturbed system.

Behavior of the velocity filter



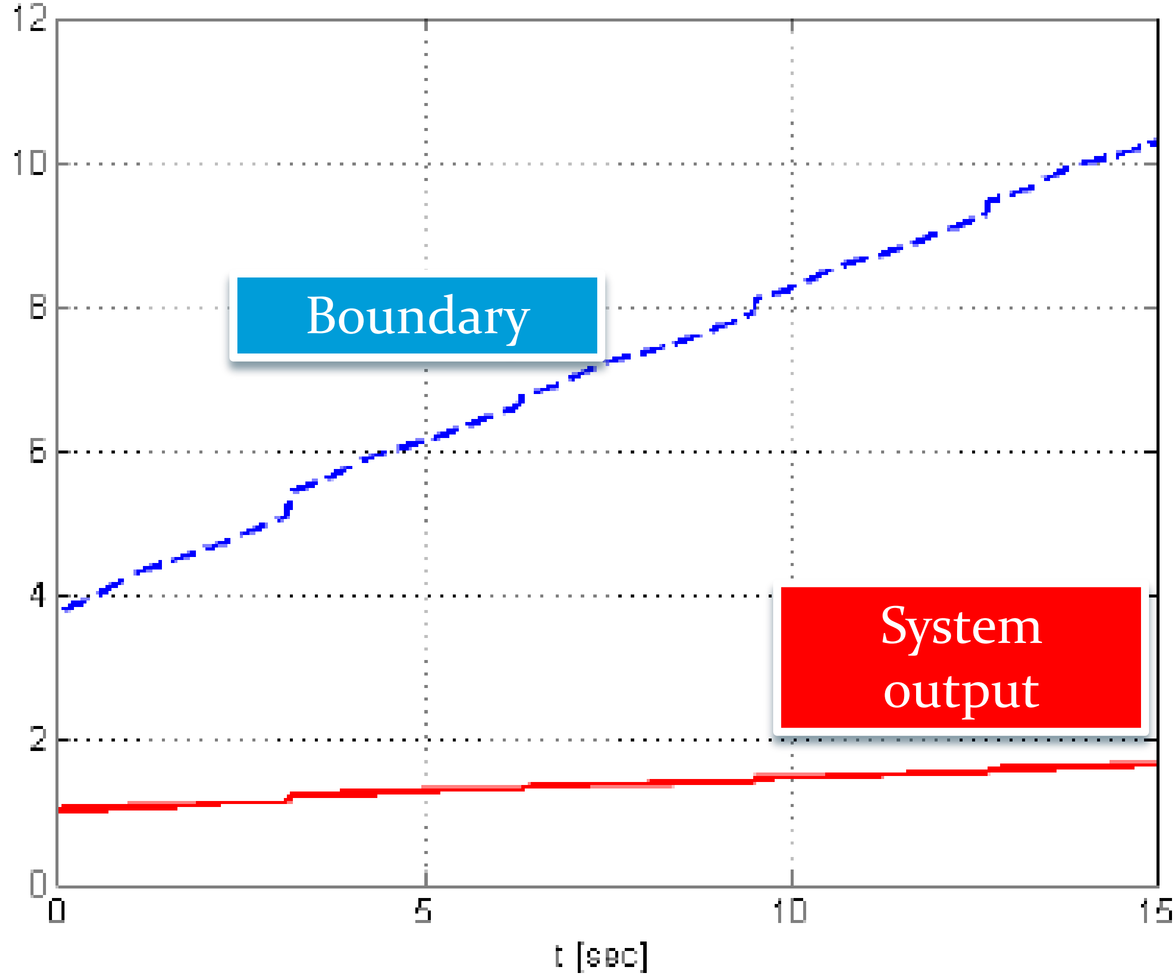


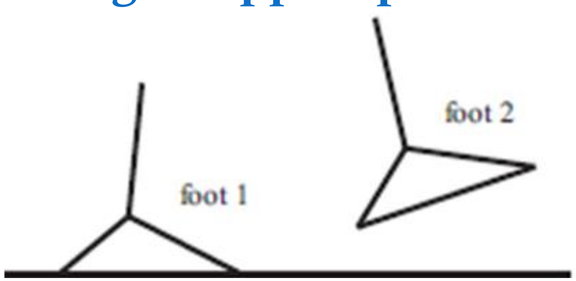
Fig. 3. $\mathcal{L}_2$-gain behavior for $\gamma = 2$: $\|z\|^2_{L_2} + \|z^d\|^2_{l_2}$ (solid line) vs. $\gamma^2[\|w\|^2_{L_2} + \|w^d_i\|^2_{l_2}] + \Sigma^N_{k=0}\beta_k$ (dashed line).

Case Study

Periodic Tracking of biped with feet via position feedback

Biped’s Hybrid Model

Single support phase



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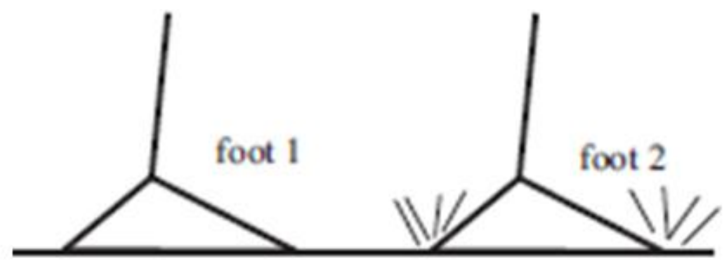
Impact phase



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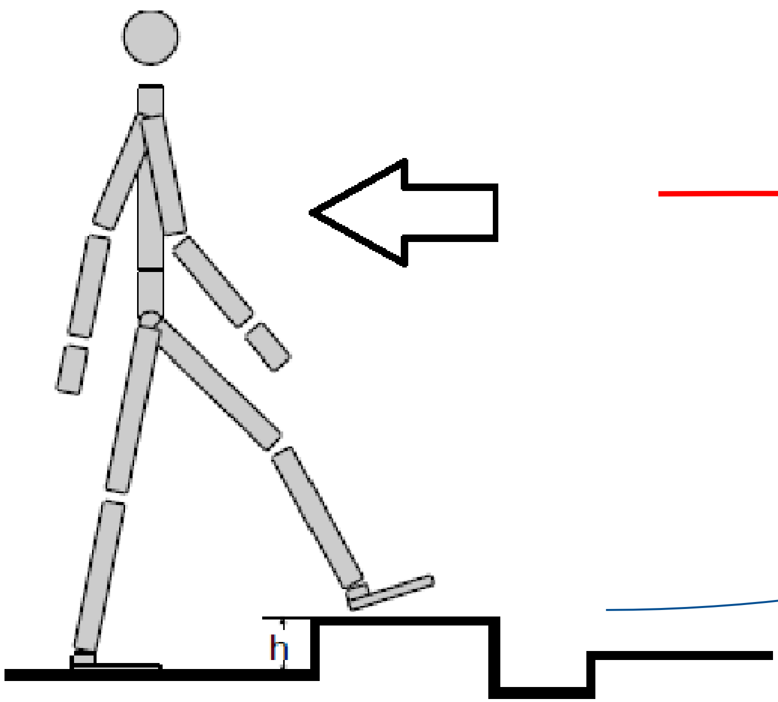
Disturbances in the model



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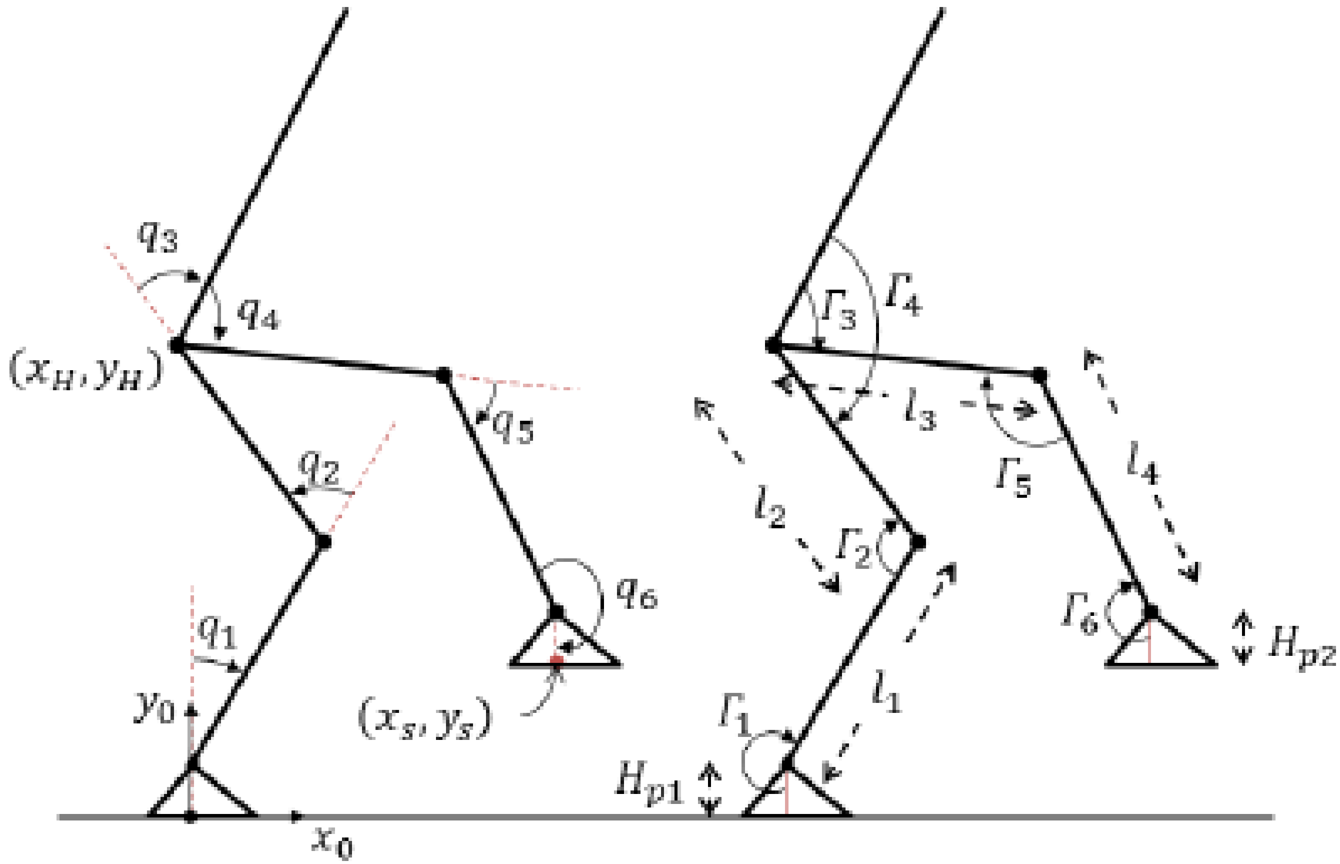
Model of the biped robot



Assumptions

- **Single support phase**:
  - Flat foot contact of the stance foot with the ground (i.e. there is no take off, no rotation, and no sliding during this phase)

- **Impact phase**
  - Flat foot contact of the swing foot with the ground, the double support phase is instantaneous and it can be modeled through passive impact equations

Model of the biped robot

- Thus, we have a model of the form

Free-motion phase:

$$(\;)\qquad (\qquad)\qquad (\;)$$
$$(\;)$$

Transition phases:

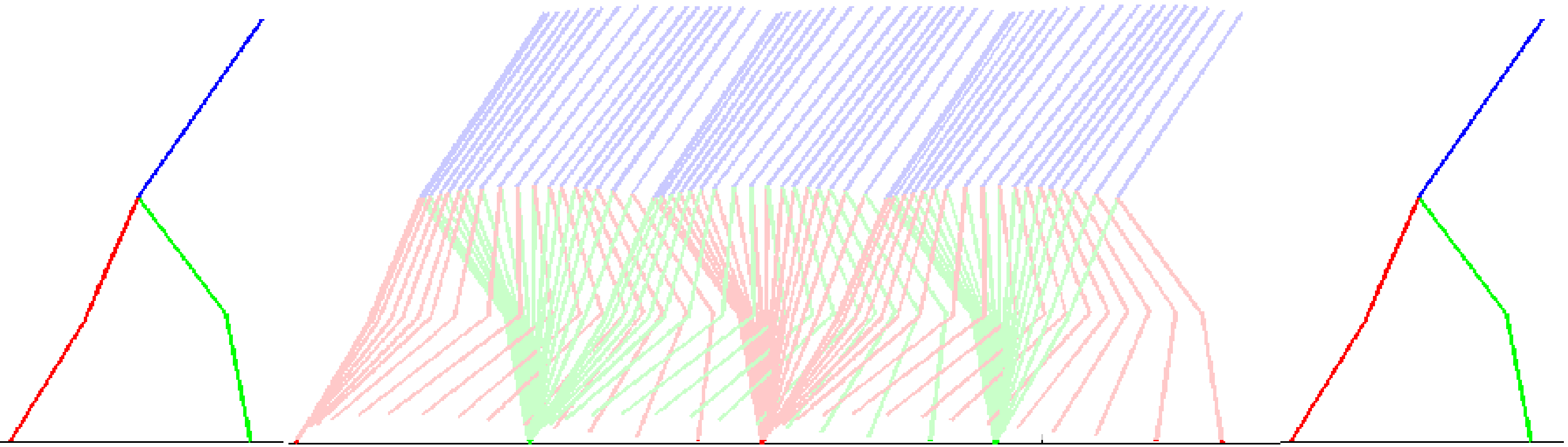
$$(\quad)\qquad (\quad)$$
$$(\quad)\qquad (\;(\quad))\; (\quad)\qquad (\quad)$$
$$(\;(\quad))$$

$(\;)$ represents the height of the swing foot

Robust control synthesis

Motion planning

Desired trajectory (   )



The trajectory:

- ❑ Minimizes an energetic criteria
- ❑ Ensures cyclic walking

Methodology

Error variables

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Prefeedback

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Error system

$$\mathbf{f}(\mathbf{x},t) = \begin{bmatrix} \mathbf{x}_2 \\ \mathbf{D}^{-1}(\mathbf{x}_1+\mathbf{q}^r)[-\mathbf{H}(\mathbf{x}_1+\mathbf{q}^r, \mathbf{x}_2+\dot{\mathbf{q}}^r) + \mathbf{H}(\mathbf{x}_1+\mathbf{q}^r, \dot{\mathbf{q}}^r)] \end{bmatrix}$$

$$\mathbf{g}_1(\mathbf{x},t) = \begin{bmatrix} \mathbf{0} \\ \mathbf{D}^{-1}(\mathbf{x}_1+\mathbf{q}^r) \end{bmatrix},$$

$$\mathbf{g}_2(\mathbf{x},t) = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix}, \ \mathbf{h}_1(\mathbf{x}) = \begin{bmatrix} \mathbf{0} \\ \rho_p \mathbf{x}_1 \\ \rho_v \mathbf{x}_2 \end{bmatrix}, \ \mathbf{k}_{12}(\mathbf{x}) = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix},$$

$$\mathbf{h}_2(\mathbf{x}) = \begin{bmatrix} \rho_3 \mathbf{I} & \mathbf{0} \end{bmatrix}, \ \mathbf{k}_{21}(\mathbf{x}) = \begin{bmatrix} \mathbf{1} & \mathbf{0} \end{bmatrix},$$

$$\boldsymbol{\mu}(\mathbf{x},t) = \begin{bmatrix} \mathbf{x}_1 \\ \phi(\mathbf{q}^r)\dot{\mathbf{q}}^r - \phi(\mathbf{x}_1+\mathbf{q}^r)(\mathbf{x}_2+\dot{\mathbf{q}}^r) \end{bmatrix},$$

$$F(\mathbf{x},t) = F_0(\mathbf{x}_1+\mathbf{q}^r), \quad \boldsymbol{\omega}(\mathbf{x},t) = -\mathbf{I}$$

Methodology

Differential Riccati Equations

$$
\begin{aligned}
-\dot{\mathbf{P}}_\varepsilon(t) &= \mathbf{P}_\varepsilon(t)\mathbf{A}(t) + \mathbf{A}^\top(t)\mathbf{P}_\varepsilon(t) + \mathbf{C_1}^\top(t)\mathbf{C_1}(t) \\
&\quad + \mathbf{P}_\varepsilon(t)[\frac{1}{\gamma^2}\mathbf{B_1}\mathbf{B_1}^\top - \mathbf{B_2}\mathbf{B_2}^\top](t)\mathbf{P}_\varepsilon(t) + \varepsilon\mathbf{I}
\end{aligned}
$$

$$
\begin{aligned}
\dot{\mathbf{Z}}_\varepsilon(t) &= \mathbf{A}_\varepsilon(t)\mathbf{Z}_\varepsilon(t) + \mathbf{Z}_\varepsilon(t)\mathbf{A}_\varepsilon^\top(t) + \mathbf{B_1}(t)\mathbf{B_1}^\top(t) \\
&\quad + \mathbf{Z}_\varepsilon(t)[\frac{1}{\gamma^2}\mathbf{P}_\varepsilon\mathbf{B_2}\mathbf{B_2}^\top\mathbf{P}_\varepsilon - \mathbf{C_2}^\top\mathbf{C_2}](t)\mathbf{Z}_\varepsilon(t) + \varepsilon\mathbf{I}
\end{aligned}
$$

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Numerical results

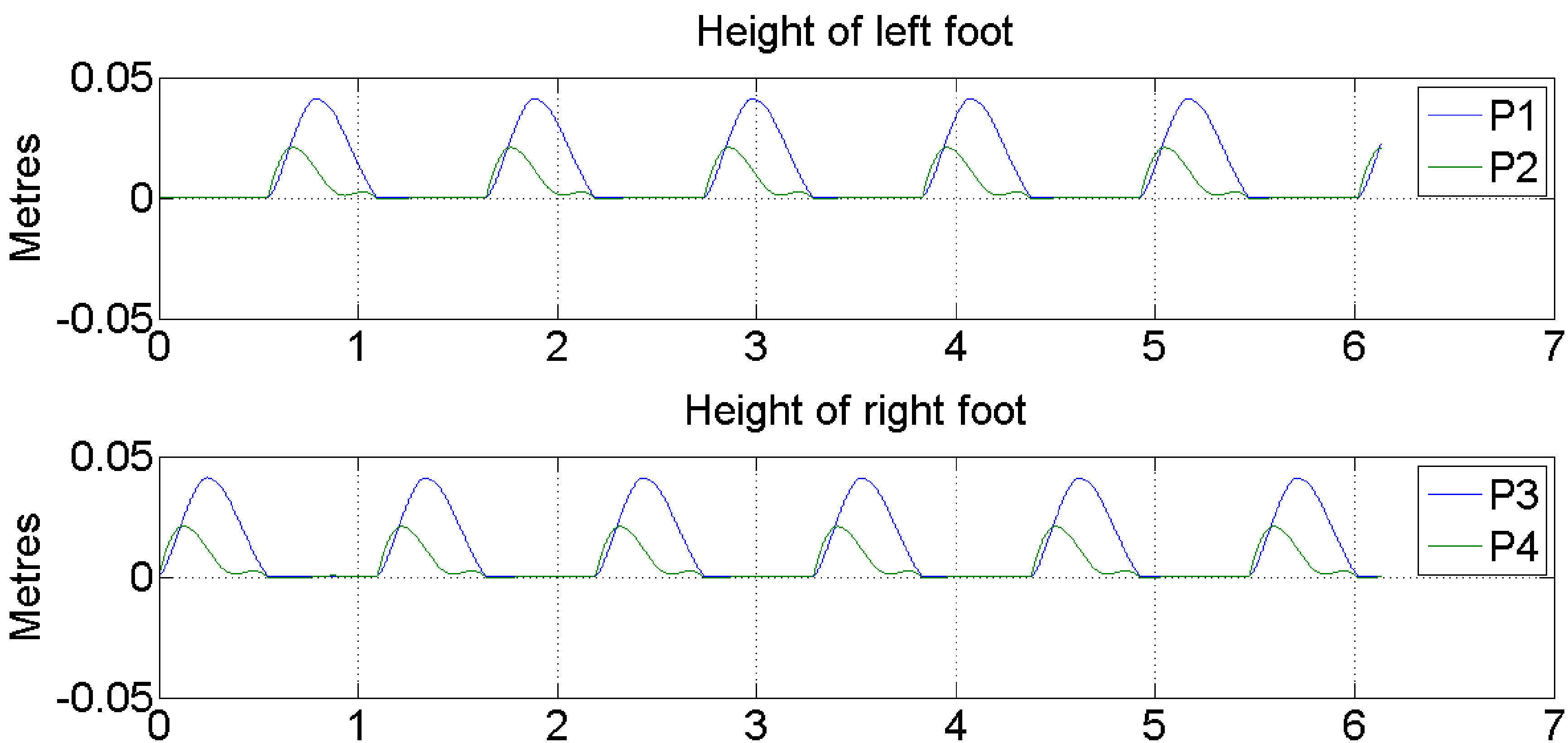
Numerical Tests

- The robustness of the tracking control is verified by introducing a disturbance force applied on the hip in the horizontal plane.
  - Such a force is used for the duration of 0.07 s to simulate a disturbance effect.

- The contact with the ground is stated as a linear complementary constraint problem and solved with a constrained optimization

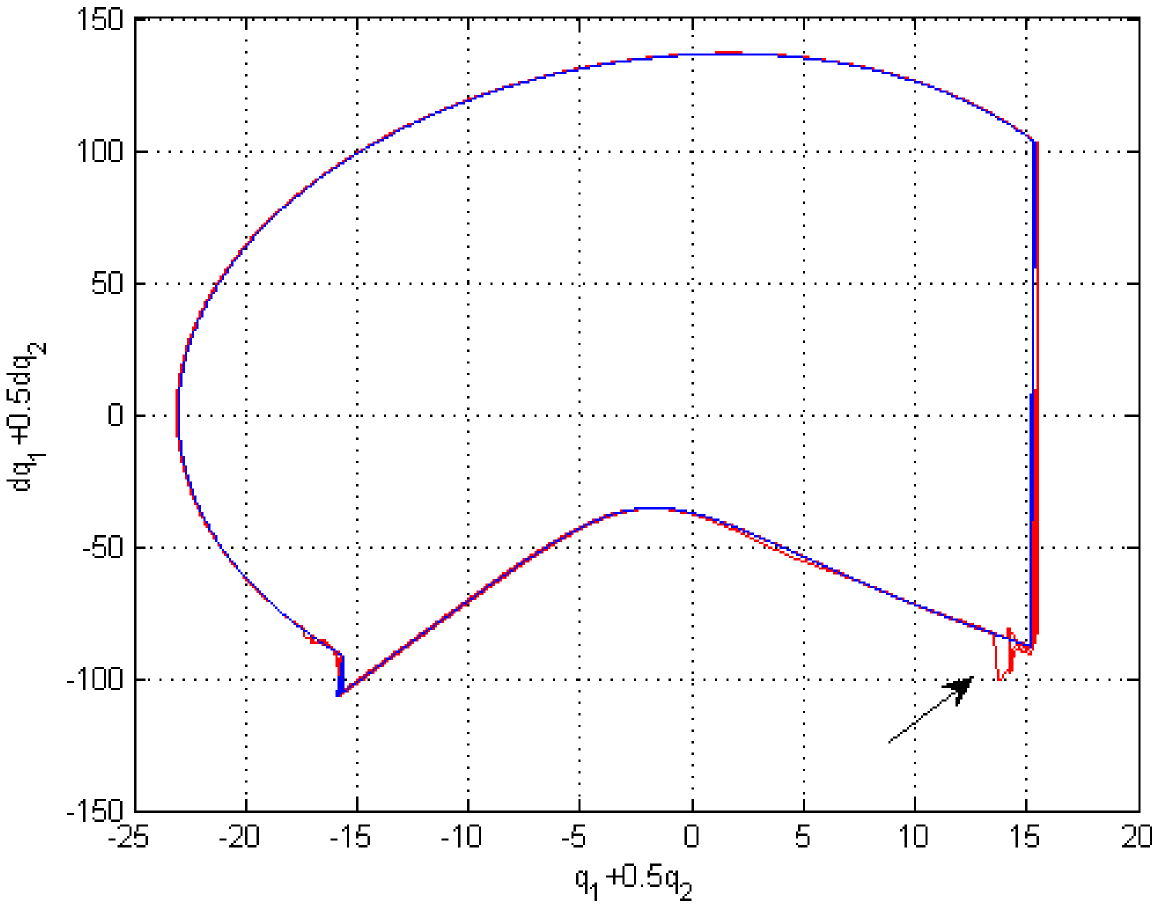
- The measurements are disturbed with a constant

Numerical results

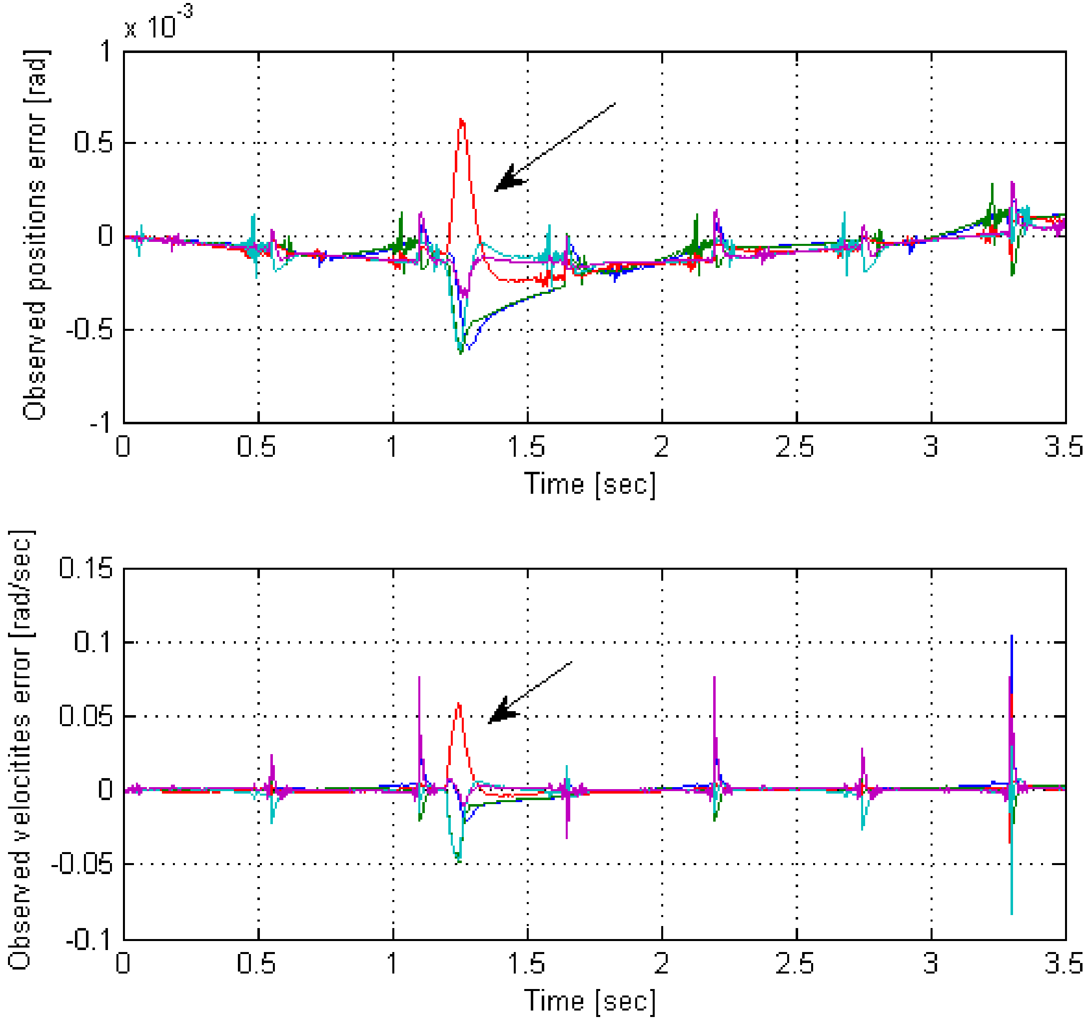


Feet heights in the walking gait

Numerical Results



Numerical Results

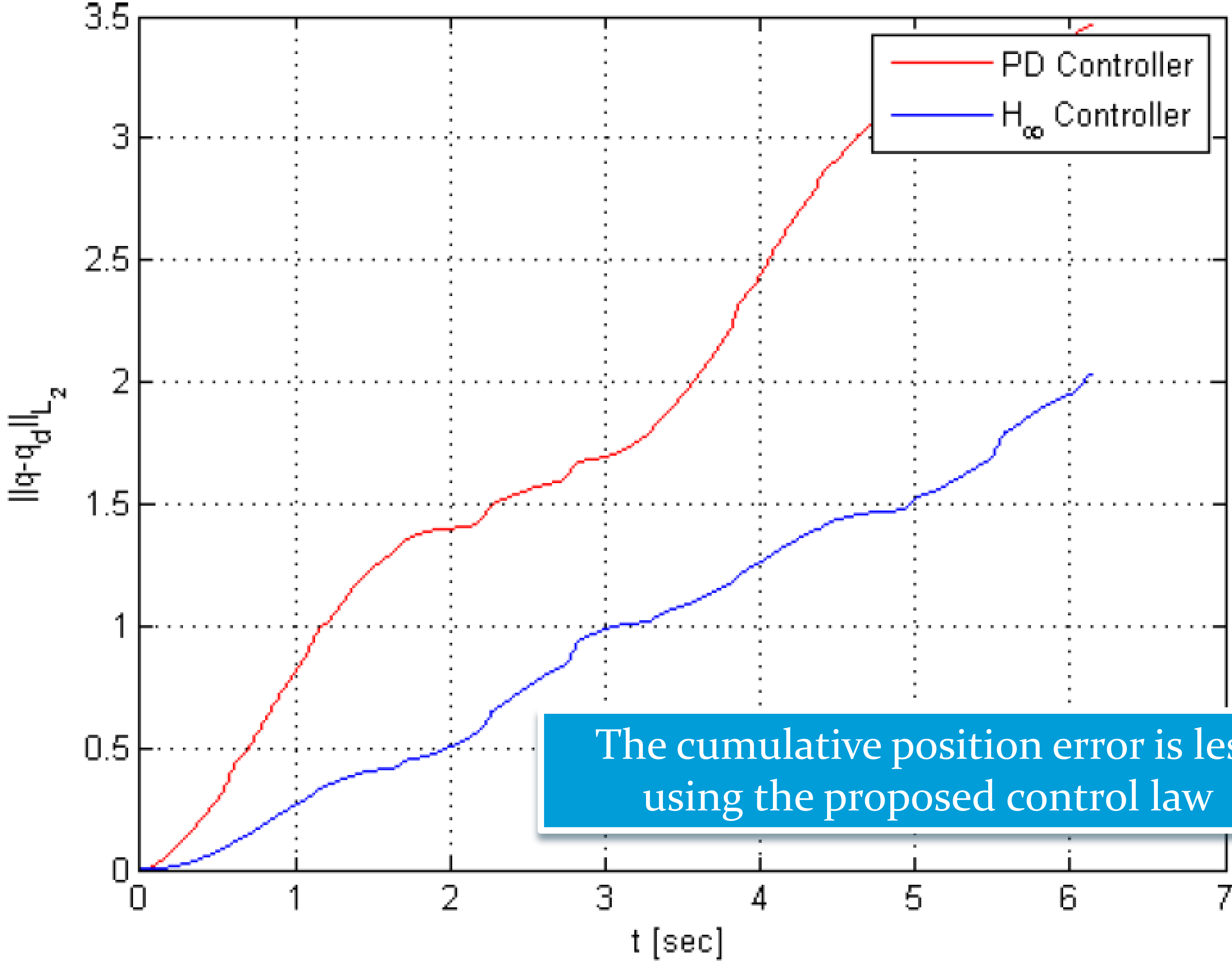


Comparisson against a PD controller

- A PD controller was designed to compare against our design

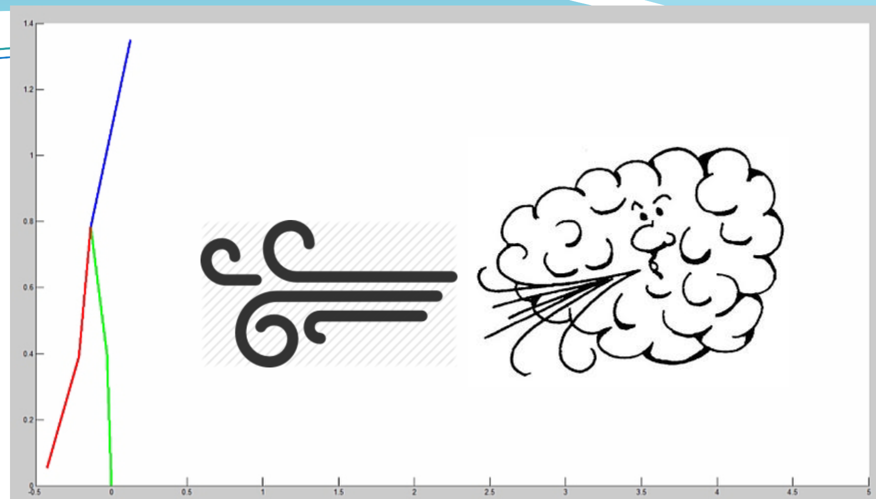
- The Parameters of the controller were selected solving a pair of time-independent Riccati equations, thus making a fair comparisson

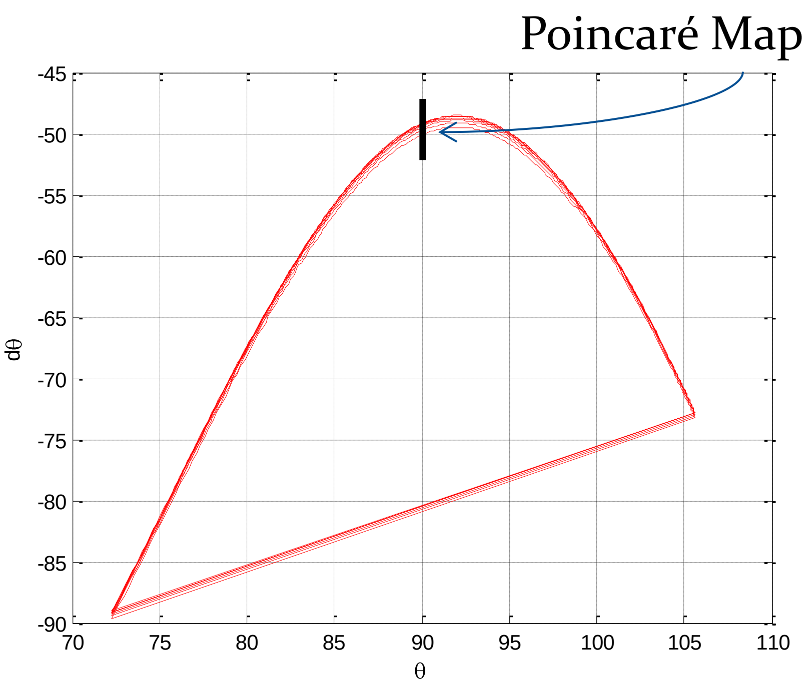
- A persisten disturbance force of
( ) was applied to the hip of the biped



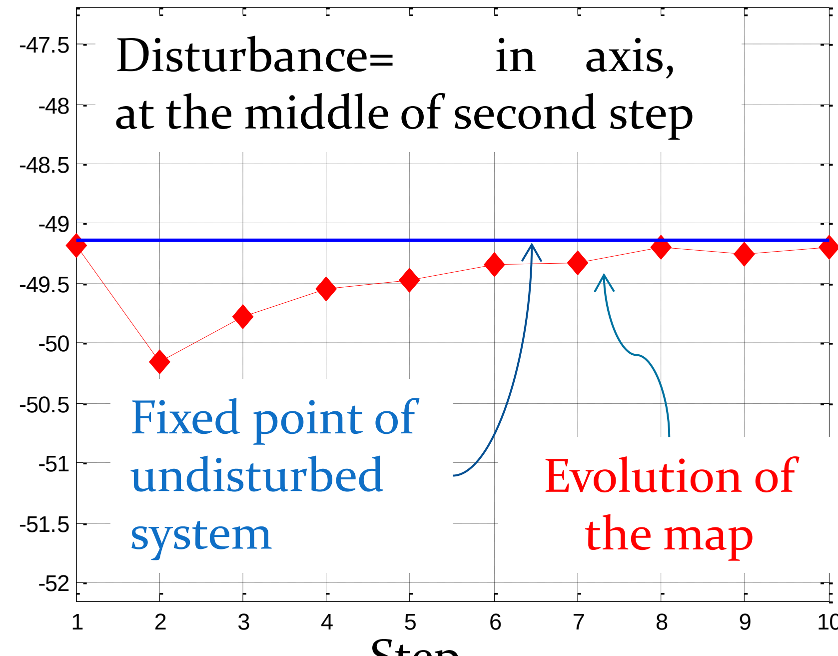
The cumulative position error is less using the proposed control law

Underactuated Biped Results





Phase plane of



Evolution of the Poincaré Map

Future work

Future work

- Extension of this results to the 3D scenario,
  - implies more degrees of freedom, thus implying more difficulties to obtain a stable walking gait.