Torque based whole body motion control for humanoid robots

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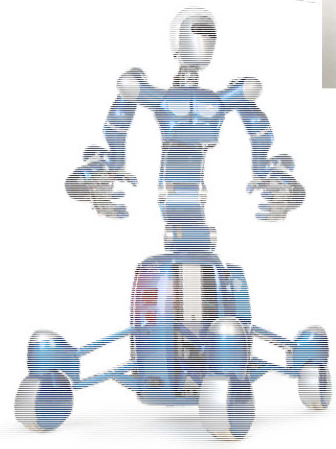
Overview

1) Control of Robots with Flexible Joints

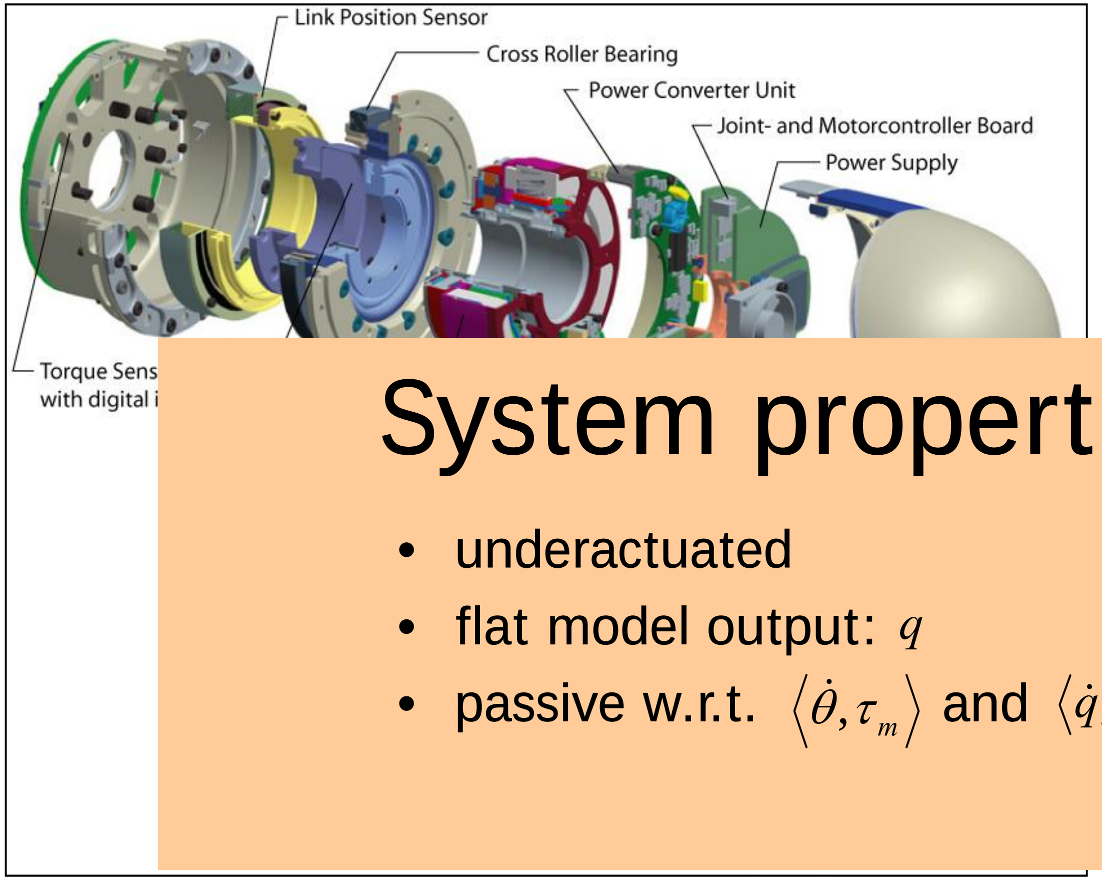
2) Multi-task compliance control

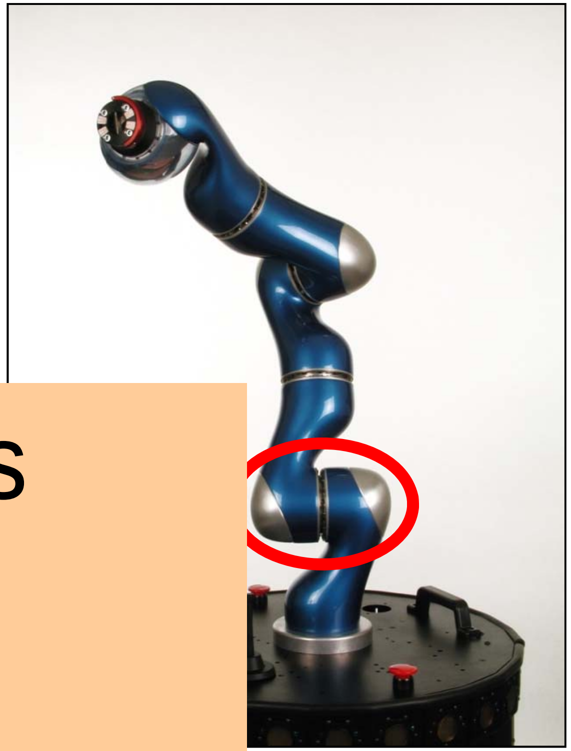
3) Extension to legged robots

4) Summary



Robot Model with Torque Sensors

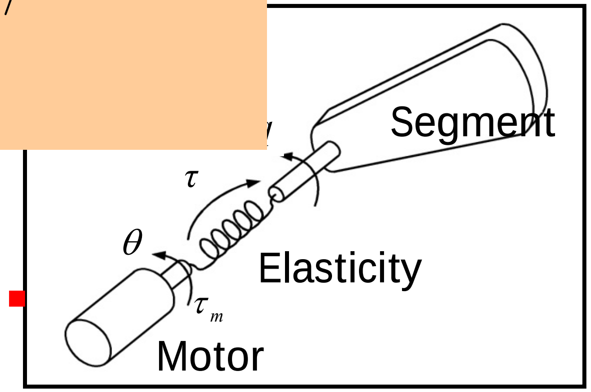




System properties

- underactuated
- flat model output: q
- passive w.r.t. $\langle \dot{\theta}, \tau_m \rangle$ and $\langle \dot{q}, \tau_{ext} \rangle$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \overbrace{K(\theta - q)}^{\tau} + \tau_{ext}$$
$$B\ddot{\theta} + K(\theta - q) = \tau_m$$

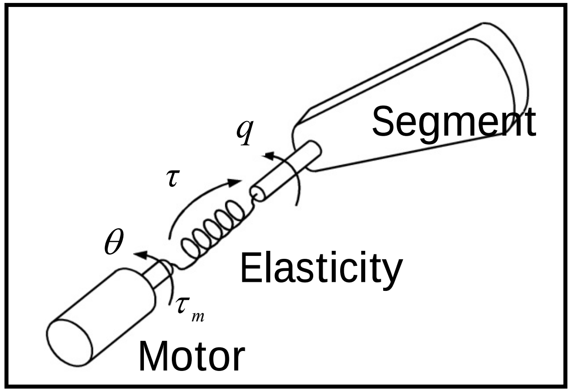


Advantages of Torque Sensing

Light-weight robot with elastic joints

Joint torque sensor

- Torque Control
- Movement accuracy
  - active vibration damping
  - Identification
- Safe human-robot-interaction
  - Disturbance Observer
  - collision reaction
  - compliance control



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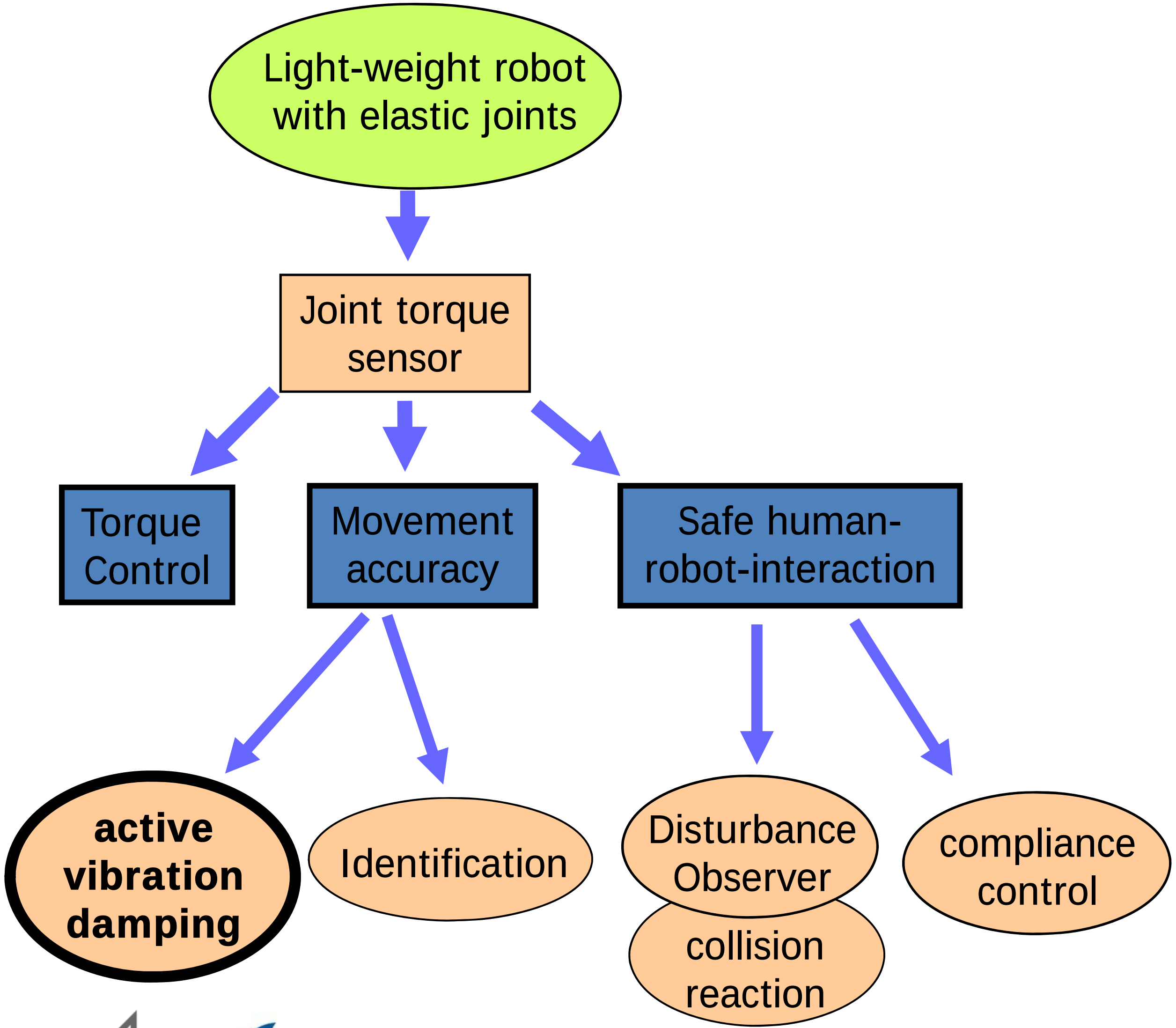
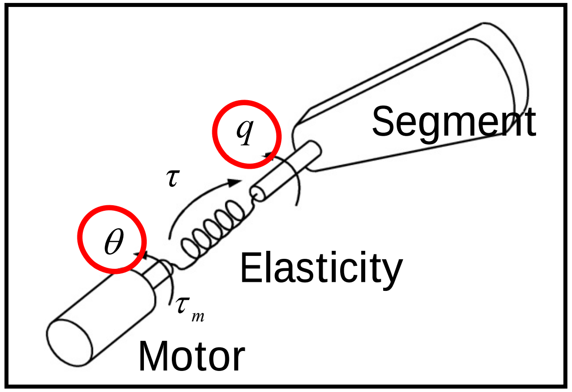
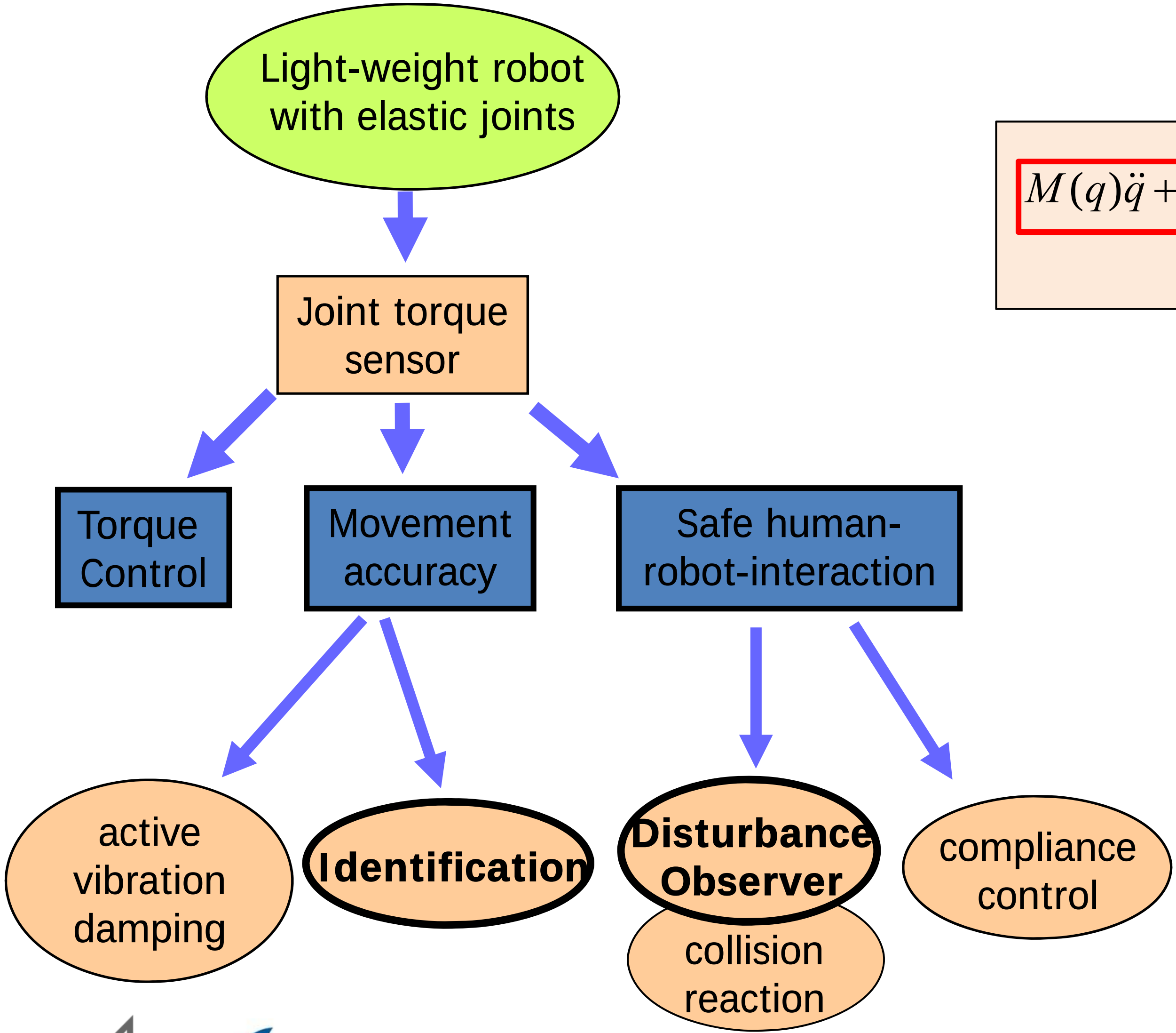
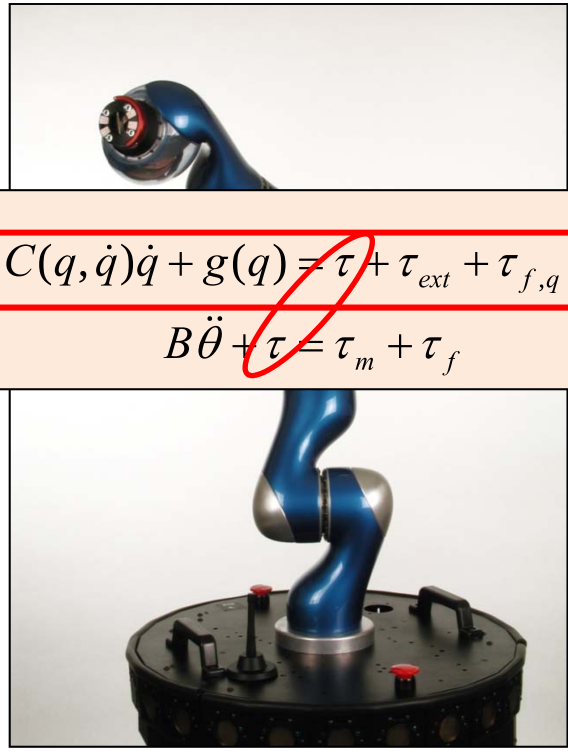




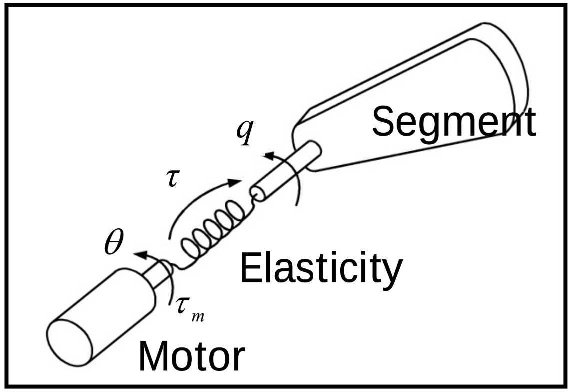
Advantages of Torque Sensing



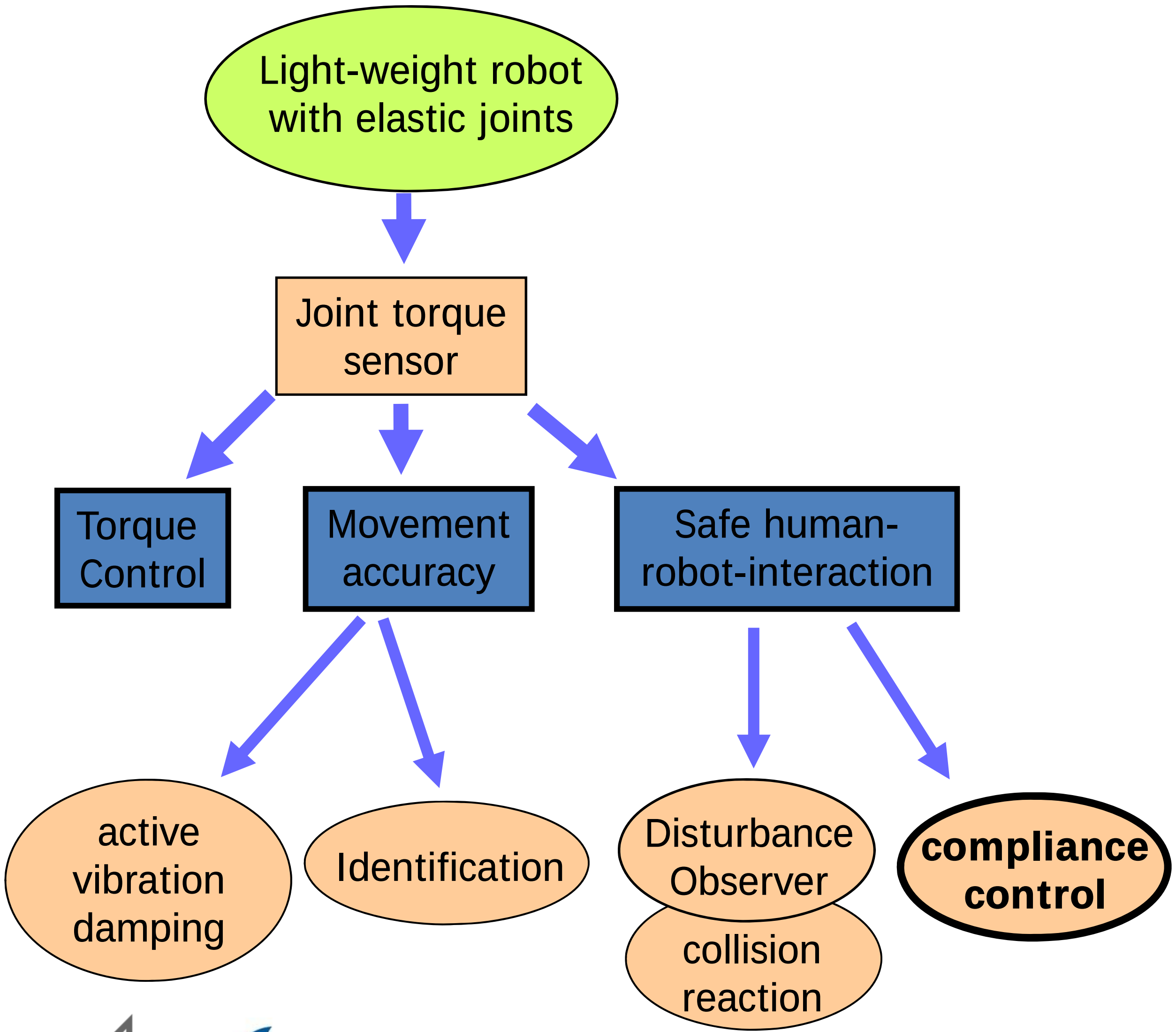


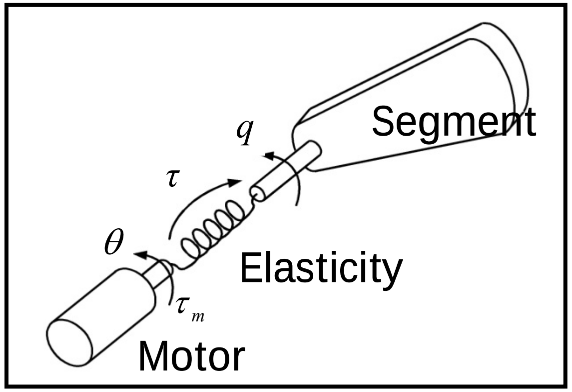
$$\boxed{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_{ext} + \tau_{f,q}}$$

$$B\ddot{\theta} + \tau = \tau_m + \tau_f$$



Advantages of Torque Sensing





Compliance Control (rigid body case)

Robot Dynamics $\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{ext}}$ $\quad \boldsymbol{q} \in \mathbb{R}^n$

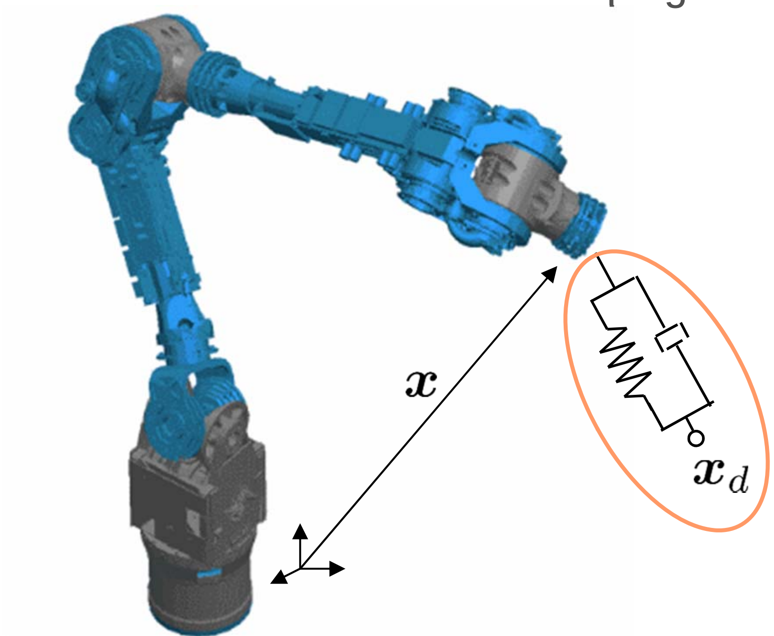
Task Space

$$\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{q}) \in \mathbb{R}^m$$

$$\dot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}$$

Control Goal (Compliance)

- Desired configuration $\boldsymbol{x}_d$
- Stiffness $\boldsymbol{K}_d \rightarrow V(\tilde{\boldsymbol{x}})$
- Damping $\boldsymbol{D}_d$ $\quad \tilde{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}_d$



Compliance Control (rigid body case)

Robot Dynamics $\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\text{ext}}$

Inverse dynamics based control

$$
\begin{aligned}
\boldsymbol{\tau} = {} & \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}(\boldsymbol{q})^T \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) \\
& + \boldsymbol{J}(\boldsymbol{q})^T \boldsymbol{\Lambda}(\boldsymbol{q}) \boldsymbol{\Lambda}_d^{-1} (\boldsymbol{K}_d(\boldsymbol{x}_d - \boldsymbol{x}) - \boldsymbol{D}_d \dot{\boldsymbol{x}}) \\
& + \boldsymbol{J}(\boldsymbol{q})^T (\boldsymbol{\Lambda}(\boldsymbol{q}) \boldsymbol{\Lambda}_d^{-1} - \boldsymbol{I}) \boldsymbol{F}_{\text{ext}}
\end{aligned}
$$

1. Exact dynamical decoupling
2. Requires external forces
3. No passive feedback

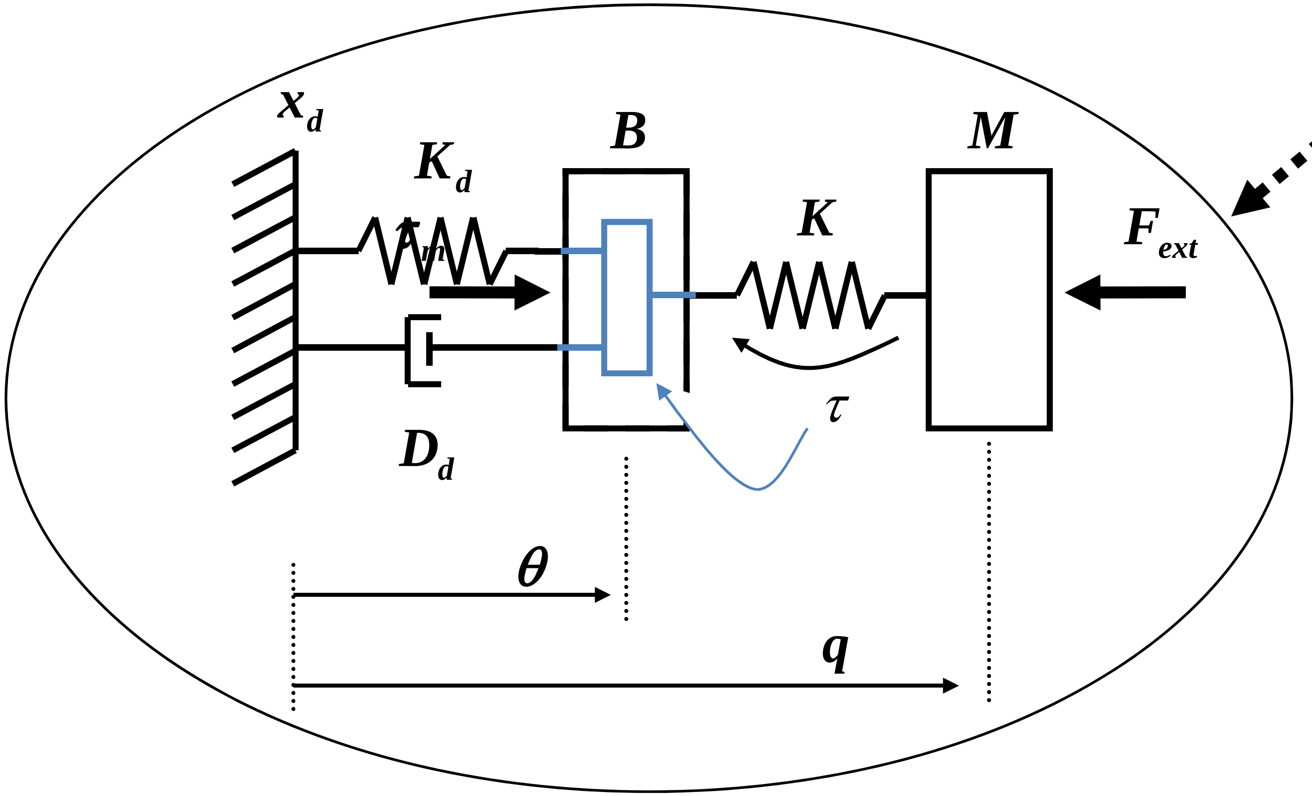
Compliance control

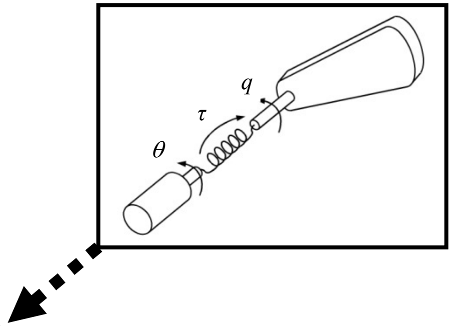
$$
\boldsymbol{\tau} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}(\boldsymbol{q})^T (\boldsymbol{K}_d(\boldsymbol{x}_d - \boldsymbol{x}) - \boldsymbol{D}_d \dot{\boldsymbol{x}})
$$

1. No inertia shaping → No feedback of external forces!
2. No cancellation of Coriolis terms
   → Passivity!
   → Physical interaction

Compliance control of elastic robots: basic idea

Consider a single elastic joint:



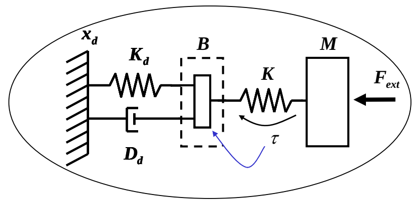


Ideally: $B \rightarrow 0$ and $K \rightarrow \infty$

Proportional feedback of the joint torque ⇔ Reduction of the motor inertia

Physical interpretation of torque control!
→ Allows for a passivity based analysis

Generalization to multi-body model



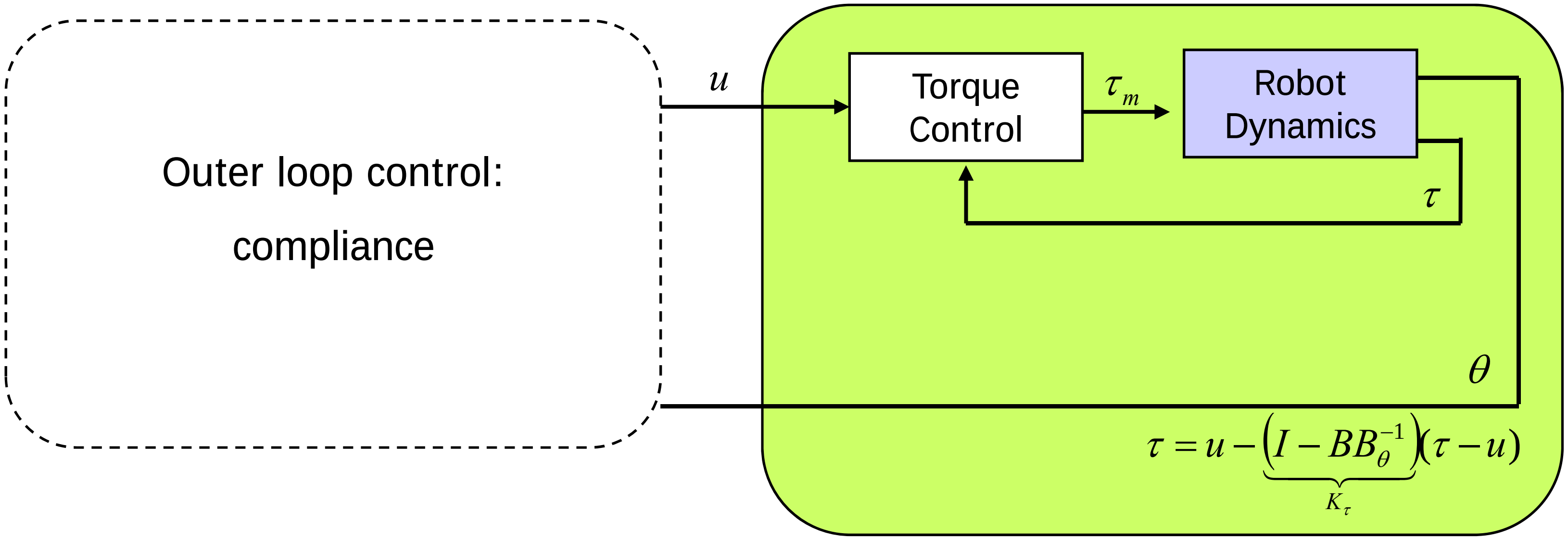
Inner loop control

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=\tau+\tau_{ext}$$
$$B\ddot{\theta}+\tau=\tau_m$$
$$\tau=K(\theta-q)$$

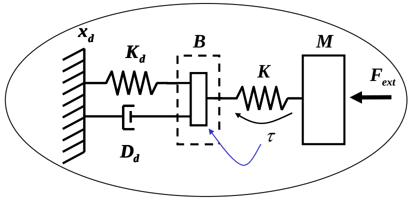
$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=\tau+\tau_{ext}$$
$$B_\theta\ddot{\theta}+\tau=u$$

$$\tau_m=\left(I-BB_\theta^{-1}\right)\tau+BB_\theta^{-1}u$$

Outer loop control for Cartesian compliance:



Generalization to multi-body model



Inner loop control

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_{ext}$$
$$B\ddot{\theta} + \tau = \tau_m$$
$$\tau = K(\theta - q)$$

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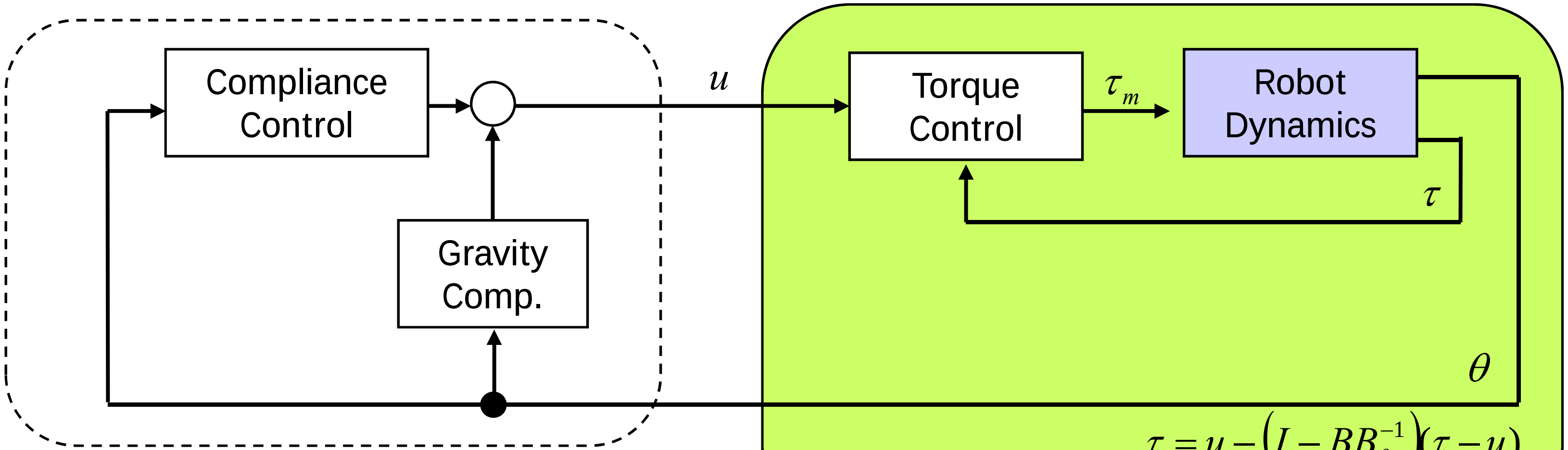
Outer loop control for Cartesian compliance:

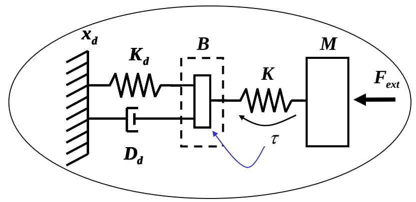


$$\tau = u - \underbrace{\left(I - BB_\theta^{-1}\right)}_{K_\tau}(\tau - u)$$

$$u = g(\overline{q}(\theta)) - \frac{\partial V(\overline{q}(\theta))}{\partial \overline{q}} - D(\theta)\dot{\theta}$$

Gravity Comp. Stiffness Damping

Generalization to multi-body model



Inner loop control

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_{ext}$$
$$B\ddot{\theta} + \tau = \tau_m$$
$$\tau = K(\theta - q)$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_{ext}$$
$$B_\theta\ddot{\theta} + \tau = u$$

$$\tau_m = \left(I - BB_\theta^{-1}\right)\tau + BB_\theta^{-1}u$$

Oute

Equilibrium: $g(q) = K(\theta - q) + \tau_{ext}$ $\qquad \tau_{ext} = \dfrac{\partial V(q)}{\partial q}$

$$\boxed{\overline{q}(\theta)}$$

→ Compensation of the static effects of K

→ Allows to fulfill requirements on the link side accuracy!

→ Computation of $\overline{q}(\theta)$ by contraction analysis!

cs

τ

θ

$$\tau = u - \underbrace{\left(I - BB_\theta^{-1}\right)}_{K_\tau}(\tau - u)$$

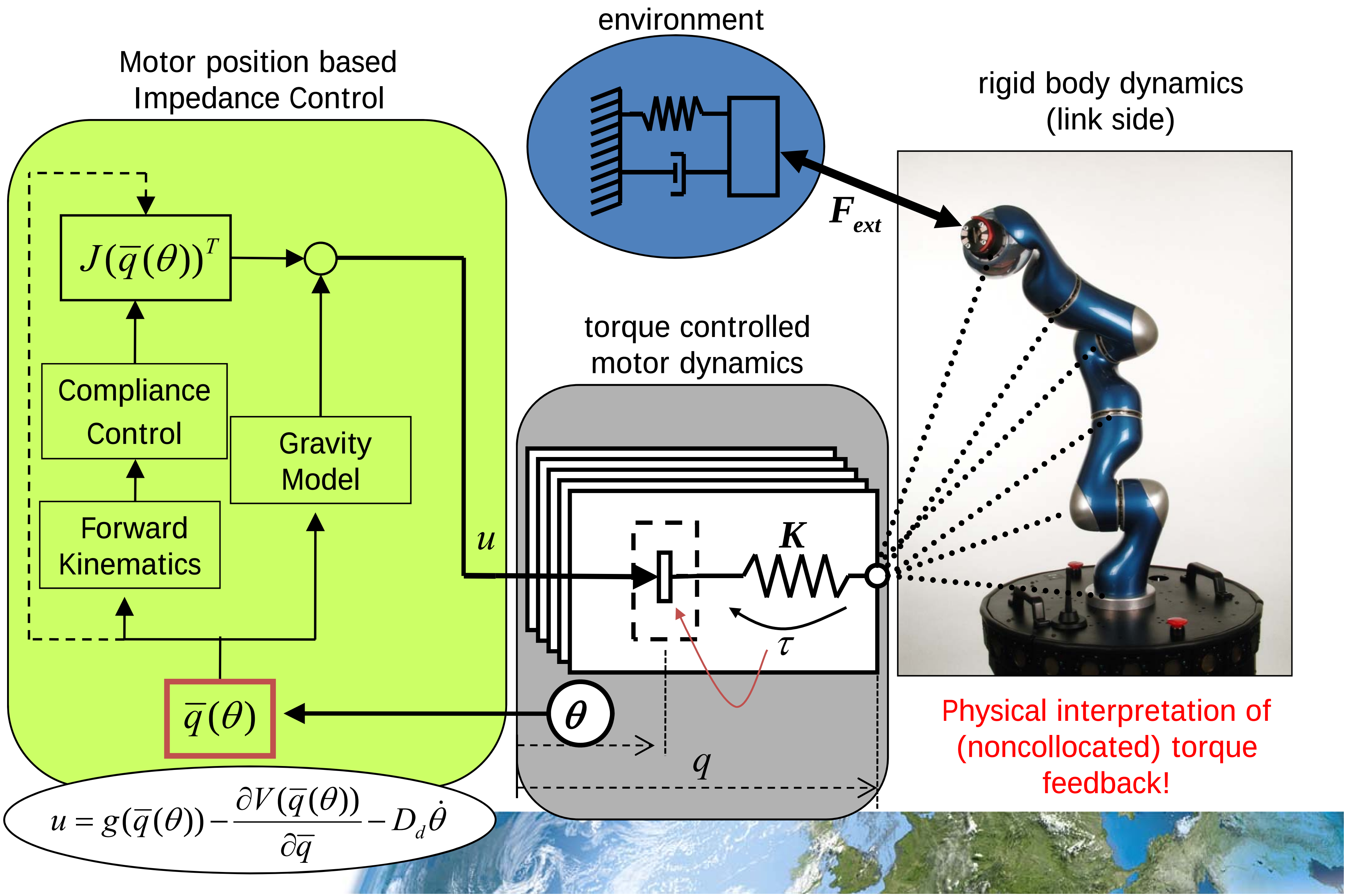
$$u = g(\overline{q}(\theta)) - \frac{\partial V(\overline{q}(\theta))}{\partial \overline{q}} - D(\theta)\dot{\theta}$$

Gravity Comp.

Stiffness

Damping

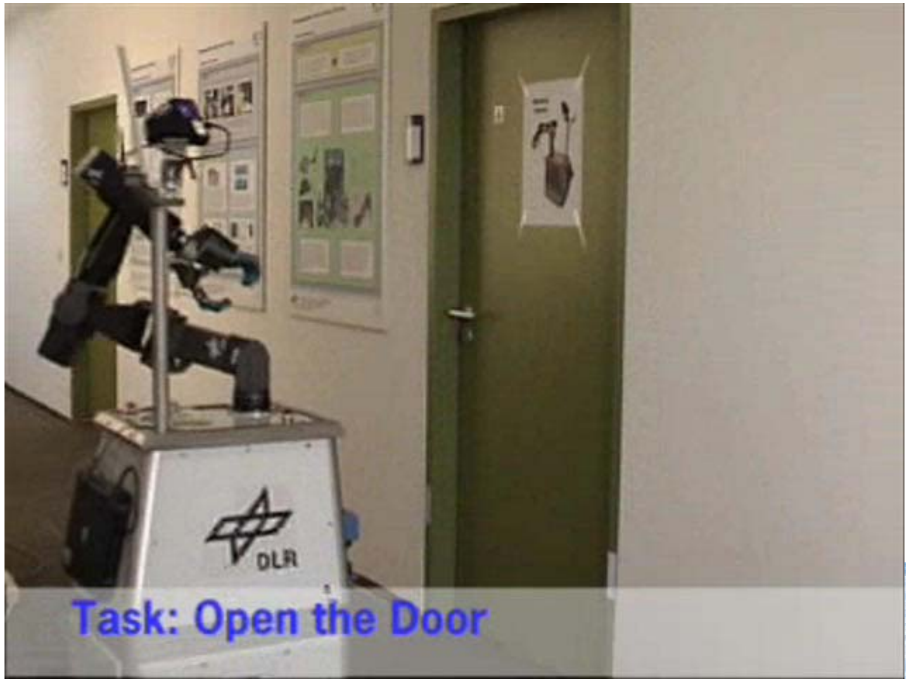
Passivity Based Control of Elastic Robots



$$u = g(\bar{q}(\theta)) - \frac{\partial V(\bar{q}(\theta))}{\partial \bar{q}} - D_d \dot{\theta}$$

Physical interpretation of (noncollocated) torque feedback!

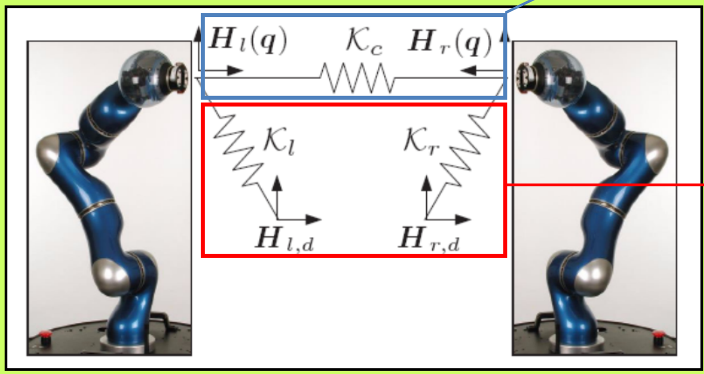
Applications (2003~2005)



Two-Armed Manipulation

Generalization of the Single Arm Impedance Control

- Control the “grasping” forces via an additional virtual spring.
- Stiffness matrices must be compatible!



$$u = g(\bar{q}(\theta)) - \frac{\partial V(\bar{q}(\theta))}{\partial \bar{q}} - D_d \dot{\theta}$$

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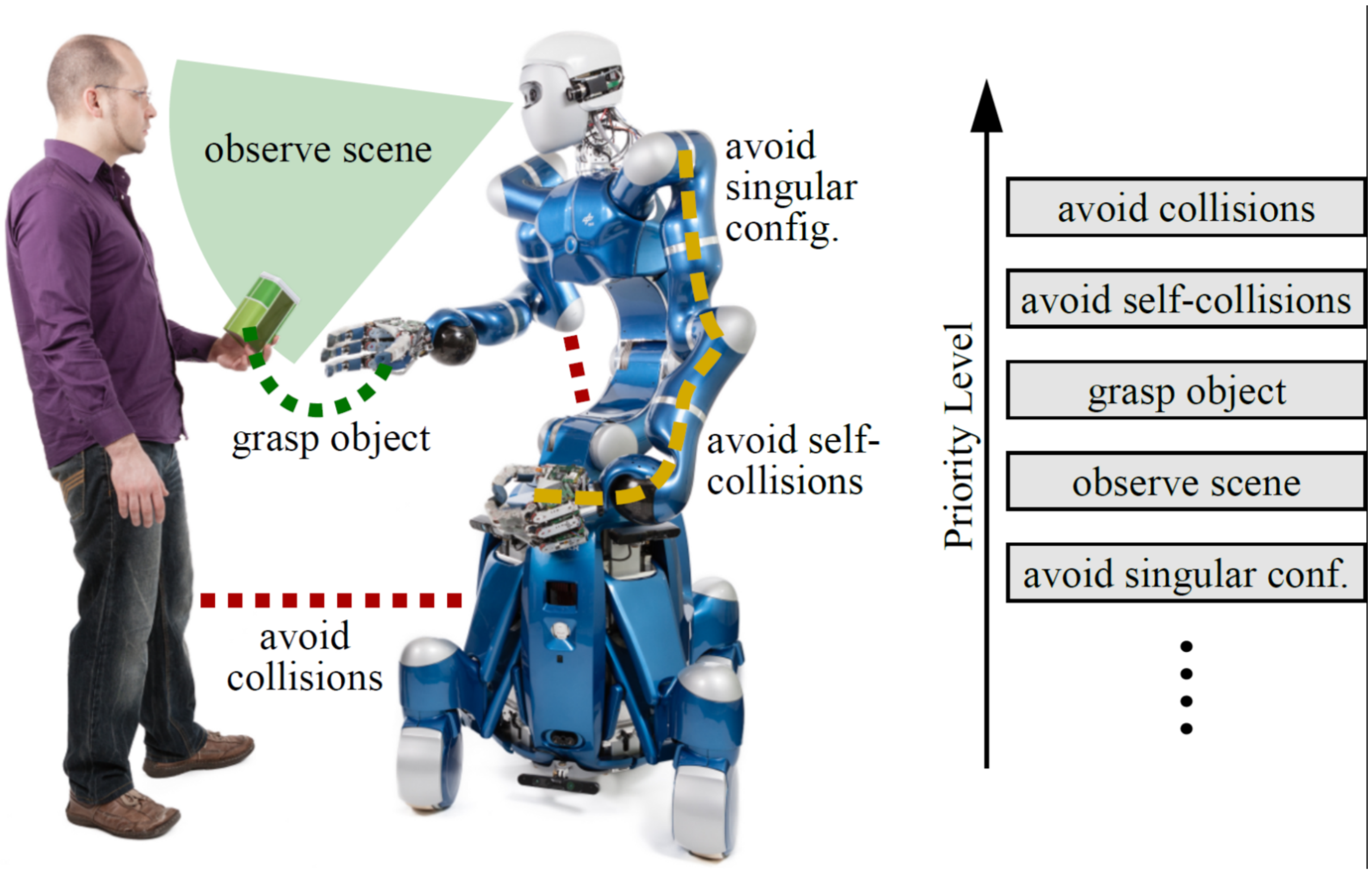
Multi-Task Control



Multi-Task Compliance Control

Robot Dynamics

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\text{ext}} \qquad \boldsymbol{q} \in \mathbb{R}^n$$

Multiple Tasks

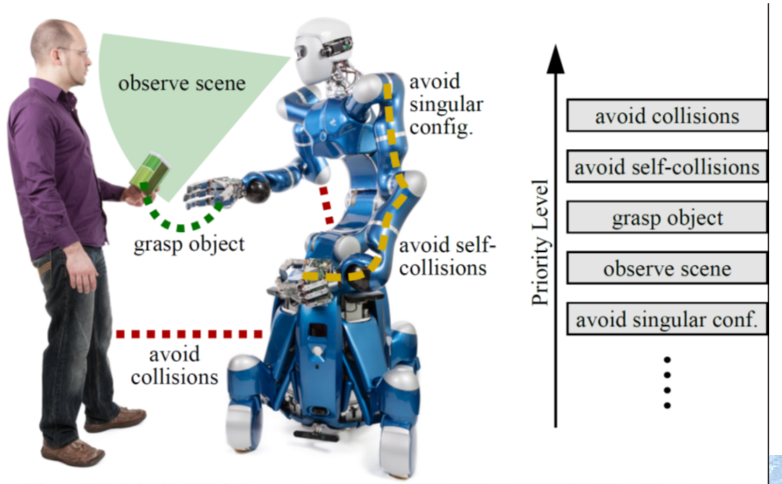
$$\boldsymbol{x}_i = \boldsymbol{f}_i(\boldsymbol{q}) \in \mathbb{R}^{m_i} \qquad \forall \; 1 \leq i \leq r$$

$$\dot{\boldsymbol{x}}_i = \boldsymbol{J}_i(\boldsymbol{q})\dot{\boldsymbol{q}}$$

$$\boldsymbol{J}_i(\boldsymbol{q}) \in \mathbb{R}^{m_i \times n} \quad \forall \; 1 \leq i \leq r$$

Control Goal (Compliance)

- Desired configurations $\boldsymbol{x}_{i,d}$
- Stiffness $\boldsymbol{K}_{i,d} \rightarrow V_i(\tilde{\boldsymbol{x}})$
- Damping $\boldsymbol{D}_{i,d}$



Task prioritization:

A task with lower priority should not disturb any task with higher priority

- execution in successive nullspaces
- successive convergence

Nullspace Compliance

Intuitive approach: nullspace projection

$$\boldsymbol{\tau} = \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{J}(\boldsymbol{q})^T \left(\frac{\partial V(\tilde{\boldsymbol{x}})}{\boldsymbol{x}} + \boldsymbol{D}_d \dot{\boldsymbol{x}} \right) + \boldsymbol{N}(\boldsymbol{q})^T \boldsymbol{\tau}_n$$

nullspace projection nullspace control action

Nullspace Compliance Control: $\boldsymbol{\tau}_n = -\frac{\partial V_n(\boldsymbol{q})}{\partial \boldsymbol{q}} - \boldsymbol{D}\dot{\boldsymbol{q}}$

For analysis:

- Cartesian dynamics allown is not enough.
- We need a dynamics formulation of the task space and nullspace
- Nullspace projection is not passive
  → above control law is not sufficient!

Nullspace Coordinates

Two possible approaches …

Additional task coordinates
(Baillieul '85)

$$x = f(q) \in \Re^{m}$$

$$n = n(q) \in \Re^{n-m}$$

Disadvantage: additional algorithmic singularities
Dynamical couplings → Problem for prioritized control

Velocity coordinates
(Park '99)

$$\begin{pmatrix} \dot{x} \\ v \end{pmatrix} = \underbrace{\begin{bmatrix} J(q) \\ N(q) \end{bmatrix}}_{J_N(q)} \dot{q} \longrightarrow \dot{q} = J_N(q)^{-1} \begin{pmatrix} \dot{x} \\ v \end{pmatrix}$$

The problem of non-integrability: Notice that in general there do not exist any nullspace position coordinates $n(q)$ such that $\partial n(q) / \partial q = N(q)$ holds.

How to choose N(q)?

Complete dynamics (Cartesian + nullspace)

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_{ext} \qquad \begin{pmatrix} \dot{x} \\ v \end{pmatrix} = \begin{bmatrix} J(q) \\ N(q) \end{bmatrix} \dot{q}$$

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$$\boxed{\Lambda_N(q)\begin{pmatrix} \ddot{x} \\ \dot{v} \end{pmatrix} + \mu_N(q,\dot{x},v)\begin{pmatrix} \dot{x} \\ v \end{pmatrix} = J_N(q)^{-T}\left(\tau + \tau_{ext} - g(q)\right)}$$

$$\Lambda_N(q) = \left(J_N(q)M(q)^{-1}J_N(q)^T\right)^{-1} = \begin{bmatrix} J(q)M(q)^{-1}J(q)^T & J(q)M(q)^{-1}N(q)^T \\ N(q)M(q)^{-1}J(q)^T & N(q)M(q)^{-1}N(q)^T \end{bmatrix}^{-1}$$

How to choose N(q)?

Complete dynamics (Cartesian + nullspace)

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_{ext} \qquad \begin{pmatrix} \dot{x} \\ v \end{pmatrix} = \begin{bmatrix} J(q) \\ N(q) \end{bmatrix} \dot{q}$$

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0

$$\Lambda_N(q) = \left(J_N(q)M(q)^{-1}J_N(q)^T\right)^{-1} = \begin{bmatrix} J(q)M(q)^{-1}J(q)^T & J(q)M(q)^{-1}N(q)^T \\ N(q)M(q)^{-1}J(q)^T & N(q)M(q)^{-1}N(q)^T \end{bmatrix}^{-1}$$

Choosing $N(q)$ based on a full row rank nullspace base matrix $Z(q)$:
→ (Huang & Varma '91, Chen & Walker '93), (Park '99)

$$\det\begin{bmatrix} J(q) \\ Z(q) \end{bmatrix} \neq 0 \qquad N(q) = \underbrace{\left(Z(q)M(q)Z(q)^T\right)^{-1}}_{\text{normalization}} Z(q)M(q)$$

$$J(q)Z(q)^T = 0$$

Hierarchical Nullspaces

Two priority levels

$$\begin{pmatrix} \dot{x} \\ v \end{pmatrix} = \begin{bmatrix} J(q) \\ N(q) \end{bmatrix} \dot{q}$$

$$J(q)Z(q)^T = 0$$
$$N(q) = \left(Z(q)M(q)Z(q)^T\right)^{-1} Z(q)M(q)$$

block-diagonal inertia matrix $\Lambda_N(q) = \begin{bmatrix} JM^{-1}J^T & 0 \\ 0 & NM(q)^{-1}N^T \end{bmatrix}^{-1}$

Multiple priority levels: *r* task coordinates

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_r \end{pmatrix} = \begin{bmatrix} J_1(q) \\ J_2(q) \\ \vdots \\ J_r(q) \end{bmatrix} \dot{q}$$

- obey priority order
- blockdiagonal inertia

$$\begin{pmatrix} \dot{x}_1 \\ v_2 \\ \vdots \\ v_r \end{pmatrix} = \begin{bmatrix} J_1(q) \\ \bar{J}_2(q) \\ \vdots \\ \bar{J}_r(q) \end{bmatrix} \dot{q}$$

Hierarchical Nullspaces

Hierarchical nullspace velocity coordinates

$$\begin{pmatrix} \dot{x}_1 \\ v_2 \\ \vdots \\ v_r \end{pmatrix} = \underbrace{\begin{bmatrix} J_1(q) \\ \bar{J}_2(q) \\ \vdots \\ \bar{J}_r(q) \end{bmatrix}}_{\bar{J}(q)} \dot{q}$$

$$\Lambda(q) = \begin{bmatrix} J_1 M^{-1} J_1^T & J_1 M^{-1} \bar{J}_2^T & \cdots & J_1 M^{-1} \bar{J}_r^T \\ \bar{J}_2 M^{-1} J_1^T & \bar{J}_2 M^{-1} \bar{J}_2^T & \cdots & \bar{J}_2 M^{-1} \bar{J}_r^T \\ \vdots & \vdots & \ddots & \vdots \\ \bar{J}_r M^{-1} J_1^T & \bar{J}_r M^{-1} \bar{J}_2^T & \cdots & \bar{J}_r M^{-1} \bar{J}_r^T \end{bmatrix}^{-1}$$

1) Decoupled inertia (dyn. consistency) $\bar{J}_i(q) M(q)^{-1} \bar{J}_j(q)^T = 0 \quad \forall i \neq j$

Hierarchical Nullspaces

Hierarchical nullspace velocity coordinates

$$\begin{pmatrix} \dot{x}_1 \\ v_2 \\ \vdots \\ v_r \end{pmatrix} = \underbrace{\begin{bmatrix} J_1(q) \\ \bar{J}_2(q) \\ \vdots \\ \bar{J}_r(q) \end{bmatrix}}_{\bar{J}(q)} \dot{q} \qquad \Lambda(q) = \begin{bmatrix} J_1 M^{-1} J_1^T & J_1 M^{-1} \bar{J}_2^T & \cdots & J_1 M^{-1} \bar{J}_r^T \\ \bar{J}_2 M^{-1} J_1^T & \bar{J}_2 M^{-1} \bar{J}_2^T & \cdots & \bar{J}_2 M^{-1} \bar{J}_r^T \\ \vdots & \vdots & \ddots & \vdots \\ \bar{J}_r M^{-1} J_1^T & \bar{J}_r M^{-1} \bar{J}_2^T & \cdots & \bar{J}_r M^{-1} \bar{J}_r^T \end{bmatrix}^{-1}$$

1) Decoupled inertia (dyn. consistency) $\bar{J}_i(q) M(q)^{-1} \bar{J}_j(q)^T = 0 \quad \forall i \neq j$

2) Hierarchy/prioritization:

$$J_{\text{aug},i-1}(q) M^{-1}(q) \bar{J}_i(q)^T = 0 \qquad J_{\text{aug},i} = \begin{bmatrix} J_1(q) \\ J_2(q) \\ \vdots \\ J_i(q) \end{bmatrix}$$

Hierarchical Nullspaces

Hierarchical nullspace velocity coordinates

$$\begin{pmatrix} \dot{x}_1 \\ v_2 \\ \vdots \\ v_r \end{pmatrix} = \underbrace{\begin{bmatrix} J_1(q) \\ \bar{J}_2(q) \\ \vdots \\ \bar{J}_r(q) \end{bmatrix}}_{\bar{J}(q)} \dot{q}$$

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3) Minimal dimension to fulfill the task
(do not span the full nullspace)

Hierarchical Nullspaces

Hierarchical nullspace velocity coordinates

$$\begin{pmatrix} \dot{x}_1 \\ v_2 \\ \vdots \\ v_r \end{pmatrix} = \underbrace{\begin{bmatrix} J_1(q) \\ \bar{J}_2(q) \\ \vdots \\ \bar{J}_r(q) \end{bmatrix}}_{\bar{J}(q)} \dot{q}$$

$$\Lambda(q) = \begin{bmatrix} J_1 M^{-1} J_1^{\,T} & J_1 M^{-1} \bar{J}_2^{\,T} & \cdots & J_1 M^{-1} \bar{J}_r^{\,T} \\ \bar{J}_2 M^{-1} J_1^{\,T} & \bar{J}_2 M^{-1} \bar{J}_2^{\,T} & \cdots & \bar{J}_2 M^{-1} \bar{J}_r^{\,T} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{J}_r M^{-1} J_1^{\,T} & \bar{J}_r M^{-1} \bar{J}_2^{\,T} & \cdots & \bar{J}_r M^{-1} \bar{J}_r^{\,T} \end{bmatrix}^{-1}$$

$$\bar{J}_i(q) M(q)^{-1} \bar{J}_j(q)^T = 0 \quad \forall i \neq j$$

Complete dynamics

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_{ext}$$

Hierarchical Nullspaces

Hierarchical nullspace velocity coordinates

$$\begin{pmatrix} \dot{x}_1 \\ v_2 \\ \vdots \\ v_r \end{pmatrix} = \underbrace{\begin{bmatrix} J_1(q) \\ \bar{J}_2(q) \\ \vdots \\ \bar{J}_r(q) \end{bmatrix}}_{\bar{J}(q)} \dot{q}$$

$$\Lambda(q) = \begin{bmatrix} J_1 M^{-1} J_1^{\,T} & J_1 M^{-1} \bar{J}_2^{\,T} & \cdots & J_1 M^{-1} \bar{J}_r^{\,T} \\ \bar{J}_2 M^{-1} J_1^{\,T} & \bar{J}_2 M^{-1} \bar{J}_2^{\,T} & \cdots & \bar{J}_2 M^{-1} \bar{J}_r^{\,T} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{J}_r M^{-1} J_1^{\,T} & \bar{J}_r M^{-1} \bar{J}_2^{\,T} & \cdots & \bar{J}_r M^{-1} \bar{J}_r^{\,T} \end{bmatrix}^{-1}$$

$$\bar{J}_i(q) M(q)^{-1} \bar{J}_j(q)^T = 0 \quad \forall i \neq j$$

Complete dynamics

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_{ext}$$

$$\Lambda(q) \begin{pmatrix} \ddot{x} \\ \dot{v}_2 \\ \vdots \\ \dot{v}_r \end{pmatrix} + \mu(q,\dot{x},v_i) \begin{pmatrix} \dot{x} \\ v_2 \\ \vdots \\ v_r \end{pmatrix} = \bar{J}(q)^{-T} \left(\tau + \tau_{ext} - g(q)\right)$$

inertially decoupled

coupled

$$\mu(q,\dot{x},v_i) = \Lambda(q)\left(\bar{J}(q)M(q)^{-1}C(q,q) - \dot{\bar{J}}(q)\right)\bar{J}(q)^{-1}$$

Controller Design

Intuitive approach: nullspace projection

$$\boldsymbol{\tau} = \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{J}(\boldsymbol{q})^T\left(\frac{\partial V(\tilde{\boldsymbol{x}})}{\boldsymbol{x}} + \boldsymbol{D}_d\dot{\boldsymbol{x}}\right) + \boldsymbol{N}(\boldsymbol{q})^T\boldsymbol{\tau}_n$$

Prioritized hierarchical compliance control

$$\tau = g(q) + \tau_\mu - J(q)^T\left(\frac{\partial V_1(\tilde{x}_1)}{\partial x_1} + D_1\dot{x}_1\right) - \sum_{i=2..r}\overbrace{\bar{J}_i(q)^T Z_i(q)}^{\text{nullspace projection}} J_i(q)^T\left(\frac{\partial V_i(\tilde{x}_i)}{\partial x_i} + D_i\dot{x}_i\right)$$

$$\bar{J}_i(q) = \left(Z_i(q)^{M^{-1}+}\right)^T$$

Controller Design

Intuitive approach: nullspace projection

$$\boldsymbol{\tau} = \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{J}(\boldsymbol{q})^T \left(\frac{\partial V(\tilde{\boldsymbol{x}})}{\boldsymbol{x}} + \boldsymbol{D}_d \dot{\boldsymbol{x}}\right) + \boldsymbol{N}(\boldsymbol{q})^T \boldsymbol{\tau}_n$$

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power conservative decoupling of Coriolis and centrifugal coupling terms

$$\tau_\mu = \bar{J}(q)^T \begin{bmatrix} 0 & \mu_{12} & \cdots & \mu_{1r} \\ -\mu_{12} & 0 & \cdots & \mu_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ -\mu_{1r} & -\mu_{2r} & \cdots & 0 \end{bmatrix} \begin{pmatrix} \dot{x}_1 \\ v_2 \\ \vdots \\ v_r \end{pmatrix}$$

$$\mu(q, \dot{x}, v_i) = \Lambda(q)\left(\bar{J}(q) M(q)^{-1} C(q, q) - \dot{\bar{J}}(q)\right)\bar{J}(q)^{-1}$$

Closed Loop Dynamics

$$\Lambda(q)\begin{pmatrix}\ddot{x}\\ \dot{v}_2\\ \vdots\\ \dot{v}_r\end{pmatrix}+\mu(q,\dot{x},v_i)\begin{pmatrix}\dot{x}\\ v_2\\ \vdots\\ v_r\end{pmatrix}=\bar{J}(q)^{-T}\left(\tau+\tau_{ext}-g(q)\right)$$

$$\tau_{ext}=\bar{J}(q)^T\begin{pmatrix}F_1\\ F_2\\ \vdots\\ F_r\end{pmatrix}$$

Main task

$$\Lambda_1(q)\ddot{x}+\mu_{11}(q,\dot{x},v_i)\dot{x}+D_d\dot{x}+\frac{\partial V_1(\tilde{x}_1)}{\partial x_1}=F_1$$

Prioritized nullspace tasks

$$\Lambda_i(q)\dot{v}_i+\mu_{ii}(q,\dot{x},v_i)v_i+Z_i(q)J_i(q)^T\left(D_d\dot{x}+\frac{\partial V_i(\tilde{x}_i)}{\partial x_i}\right)=F_i$$

Iterative Convergence

Main Task: $$\boxed{\Lambda_1(q)\ddot{x} + \mu_{11}(q,\dot{x},v_i)\dot{x} + D_d\dot{x} + \frac{\partial V_1(\tilde{x}_1)}{\partial x_1} = F_1}$$

$$S_1 = \frac{1}{2}\dot{x}^T\Lambda_1(q)\dot{x} + V_1(\tilde{x}_1)$$

$$\dot{S}_1 = \dot{x}^T F_1 - \dot{x}^T D_1\dot{x}$$

Passivity, convergence

Iterative Convergence

Main Task:

$$\boxed{\Lambda_1(q)\ddot{x} + \mu_{11}(q,\dot{x},v_i)\dot{x} + D_d\dot{x} + \frac{\partial V_1(\tilde{x}_1)}{\partial x_1} = F_1}$$

$$S_1 = \frac{1}{2}\dot{x}^T\Lambda_1(q)\dot{x} + V_1(\tilde{x}_1)$$

$$\dot{S}_1 = \dot{x}^T F_1 - \dot{x}^T D_1\dot{x}$$

Passivity, convergence

Subtasks:

$$\boxed{\Lambda_i(q)\dot{v}_i + \mu_{ii}(q,\dot{x},v_i)v_i + Z_i(q)J_i(q)^T\left(D_i\dot{x} + \frac{\partial V_i(\tilde{x}_i)}{\partial x_i}\right) = F_i}$$

$$S_i = \frac{1}{2}v_i^T\Lambda_i(q)v_i + V_i(\tilde{x}_i)$$

$$\dot{S}_i = v_i^T F_i - v_i^T Z_i J_i^T D_i\dot{x}_i - v_i^T Z_i J_i^T\frac{\partial V_i(\tilde{x}_i)}{\partial x_i} + \frac{\partial V_i(\tilde{x}_i)}{\partial x_i}\dot{x}_i$$

After convergence of higher levels:

Passivity & convergence of next subtask

$$\bar{J}_{\text{aug},i-1}(q)Z_i(q)^T = 0$$

After convergence of higher priority tasks: $\dot{x}_i = J_i\underbrace{Z_i v_i}_{\dot{q}}$

$$\Rightarrow \quad \dot{S}_i = v_i^T F_i - \underbrace{v_i^T Z_i J_i^T D_i J_i Z_i^T v_i}_{\leq 0}$$

Iterative Convergence

Main Task: $$\Lambda_1(q)\ddot{x} + \mu_{11}(q,\dot{x},v_i)\dot{x} + D_d\dot{x} + \frac{\partial V_1(\tilde{x}_1)}{\partial x_1} = F_1$$

$$S_1 = \frac{1}{2}\dot{x}^T \Lambda_1(q)\dot{x} + V_1(\tilde{x}_1)$$

Subtas

$= F_i$

$\tilde{x}_i)$ $\dot{x}_i$

After cor

Stability based on semi-definite Lyapunov functions

Theorem 1: [12] *Consider a system of the form $\dot{z} = f(z)$, $z \in \mathbb{R}^n$, with equilibrium point z^*. Let $V(z)$ be a C^1 positive semi-definite function which has a negative semi-definite time derivative along the solutions of the system, i.e.*

$$\dot{V}(z) = \frac{\partial V(z)}{\partial z} f(z) \leq 0\ . \qquad (13)$$

Let $\mathcal{A}$ be the largest positively invariant set contained in $\{z \in \mathbb{R}^n | V(z) = 0\}$. If z^* is asymptotically stable conditionally to $\mathcal{A}$, then it is a stable equilibrium of $\dot{z} = f(z)$.

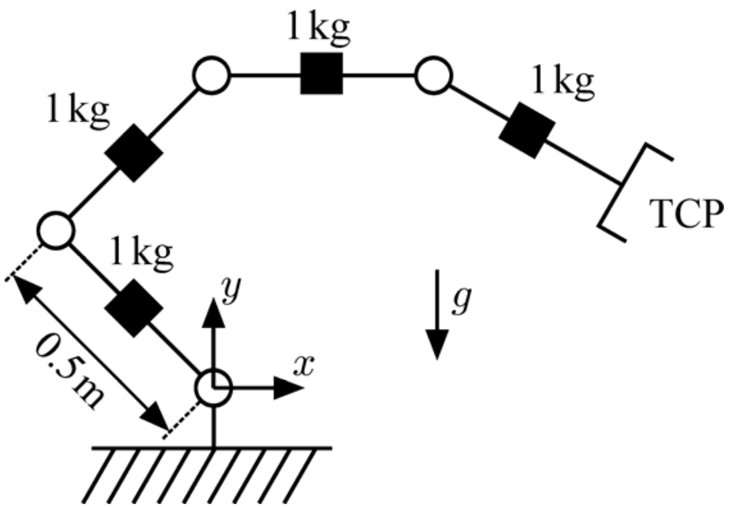
$J_{\text{aug},i-1}(q)Z_i(q)^T = 0$

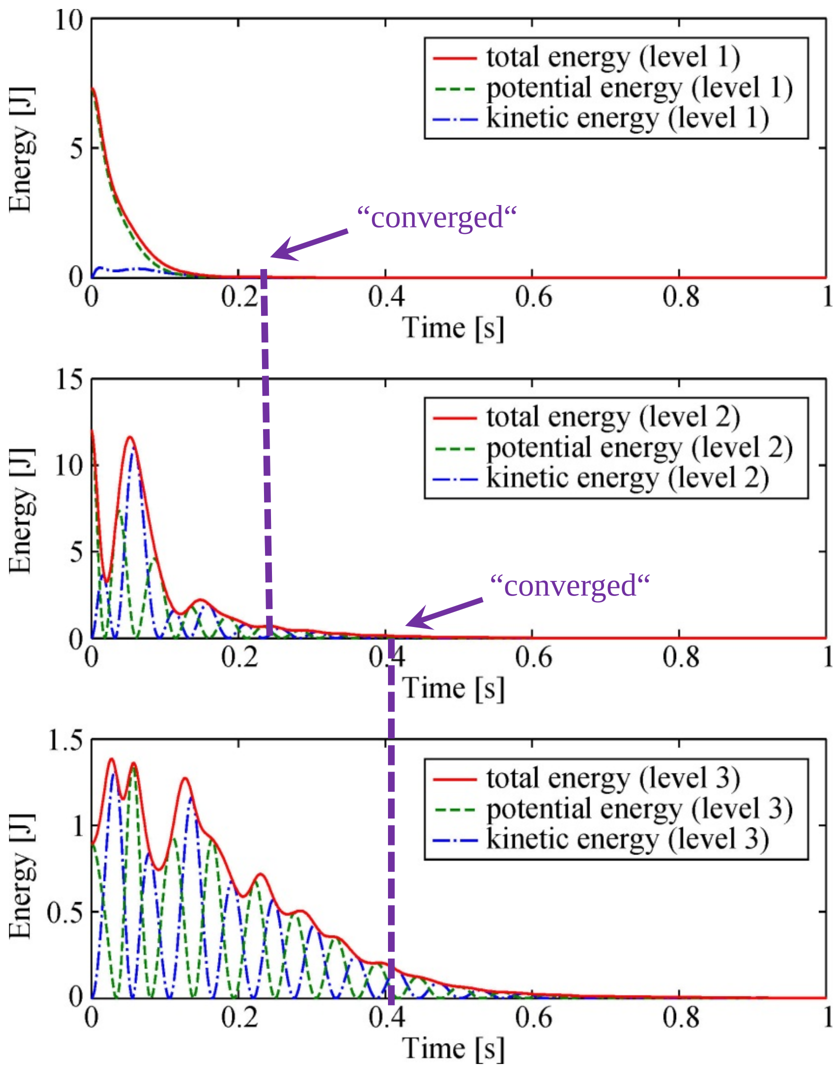
Passivity & convergence of next subtask

After convergence of higher priority tasks: $\dot{x}_i = J_i \underbrace{Z_i v_i}_{\dot{q}}$

$$\dot{S}_i = v_i^T F_i - \underbrace{v_i^T Z_i J_i^T D_i J_i Z_i^T v_i}_{\leq 0}$$

Simulations





Priority levels for simulation:

1. Translational Cartesian compliance of the TCP
2. Rotational Cartesian compliance of the TCP
3. Compliance of first joint

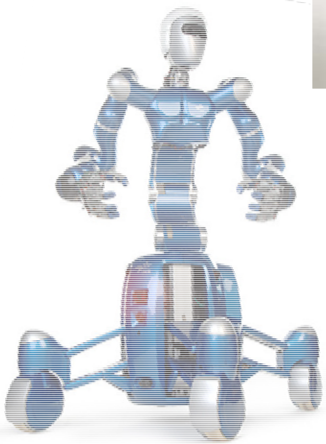
Overview

1) Control of Robots with Flexible Joints

2) Multi-task compliance control

3) Extension to legged robots

4) Summary

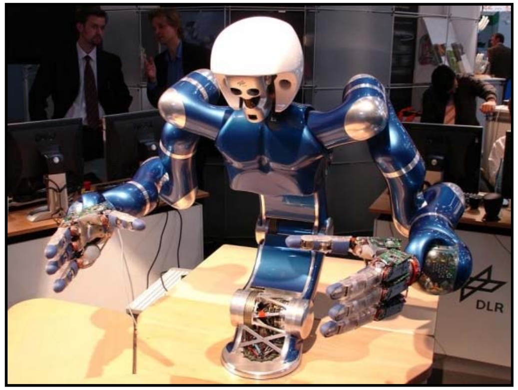


Torque controlled humanoids at DLR

- Joint torque sensing & control
- Robust manipulation via impedance control

LBR-III (2003)

Justin (2006)



Biped (2010)

TORO (2013)

Space Qualified
Joint Technology

Mobile Justin (2009)

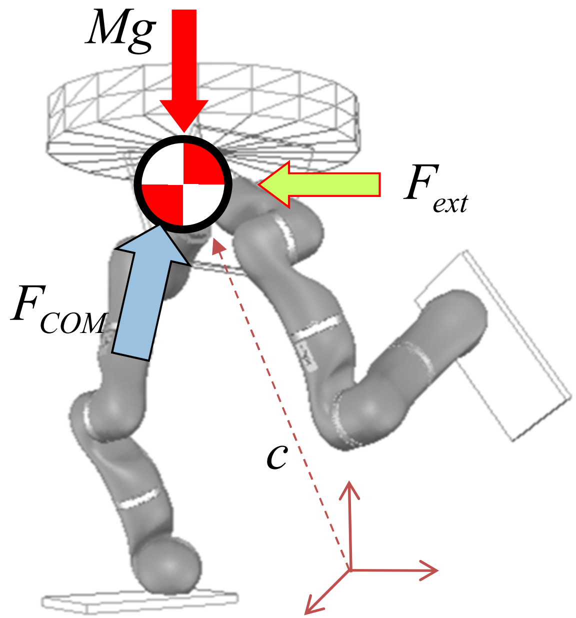
- Joint torque sensing & control
- Small foot size: 19 x 9,5 cm
- IMU in head & trunk
- FTS in feet for position based control
- Sensorized head (stereo vision & kinect)
- Simple prosthetic hands (iLIMB)

Applications

Balancing & Posture Control

Compliant COM control [Hyon & Cheng, 2006]

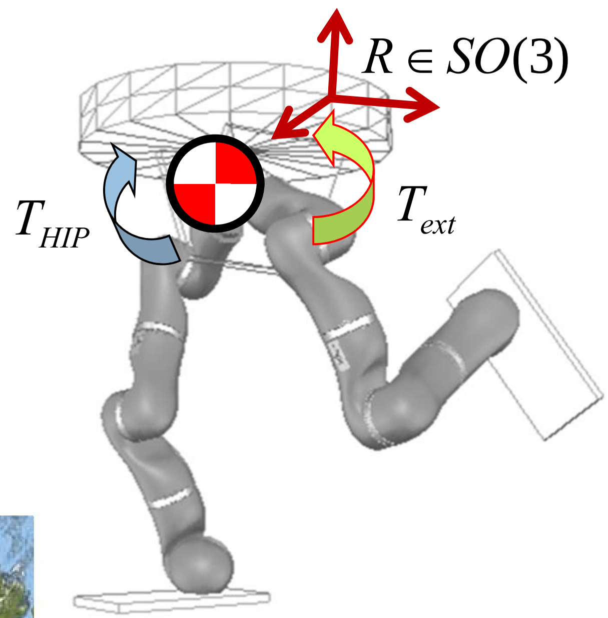
$$F_{COM} = Mg - K_P(c - c_d) - K_D(\dot{c} - \dot{c}_d)$$



Trunk orientation Control

$$T_{HIP} = \frac{\partial \dot{V}(R, K_R)}{\partial \omega} + D_R(\omega - \omega_d)$$

IMU measurements



Balancing & Posture Control

Compliant COM control [Hyon & Cheng, 2006]

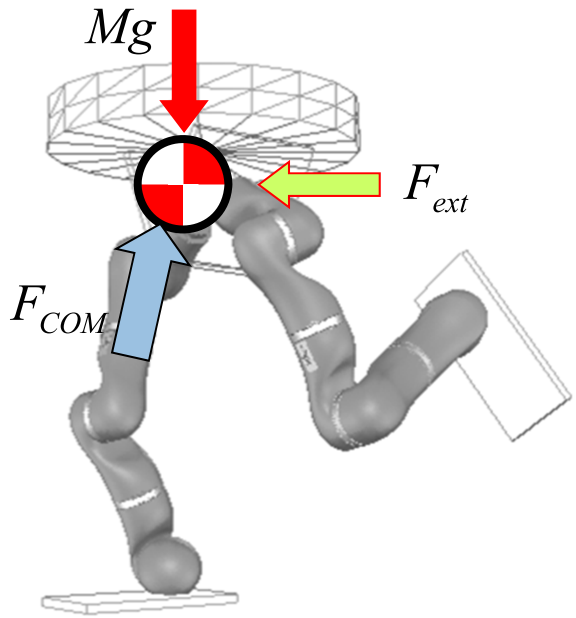
$$F_{COM} = Mg - K_P(c - c_d) - K_D(\dot{c} - \dot{c}_d)$$

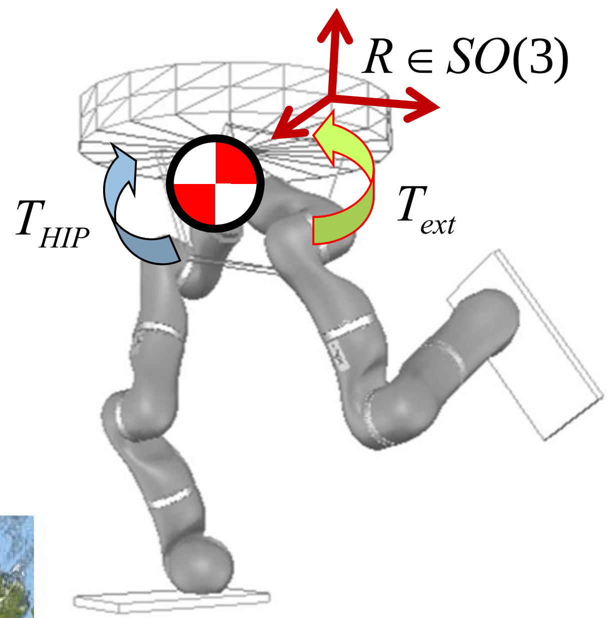
Desired wrench: $W_d = (F_{COM}, T_{HIP})$

Trunk orientation Control

$$T_{HIP} = \frac{\partial \dot{V}(R, K_R)}{\partial \omega} + D_R(\omega - \omega_d)$$

IMU measurements





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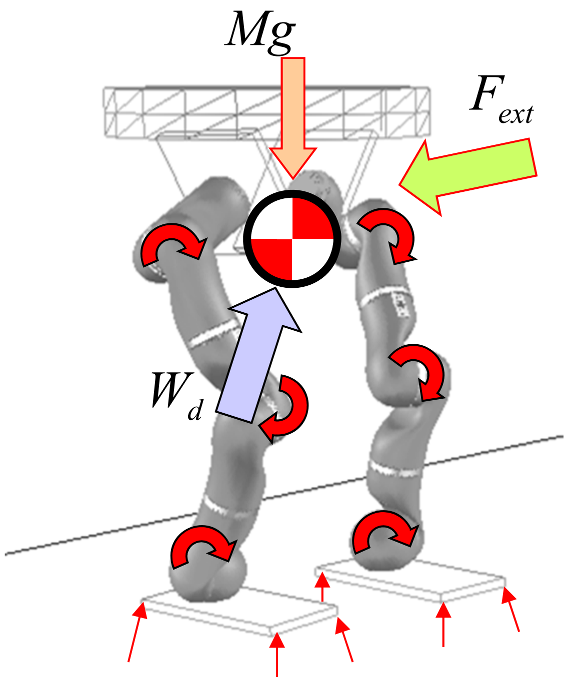
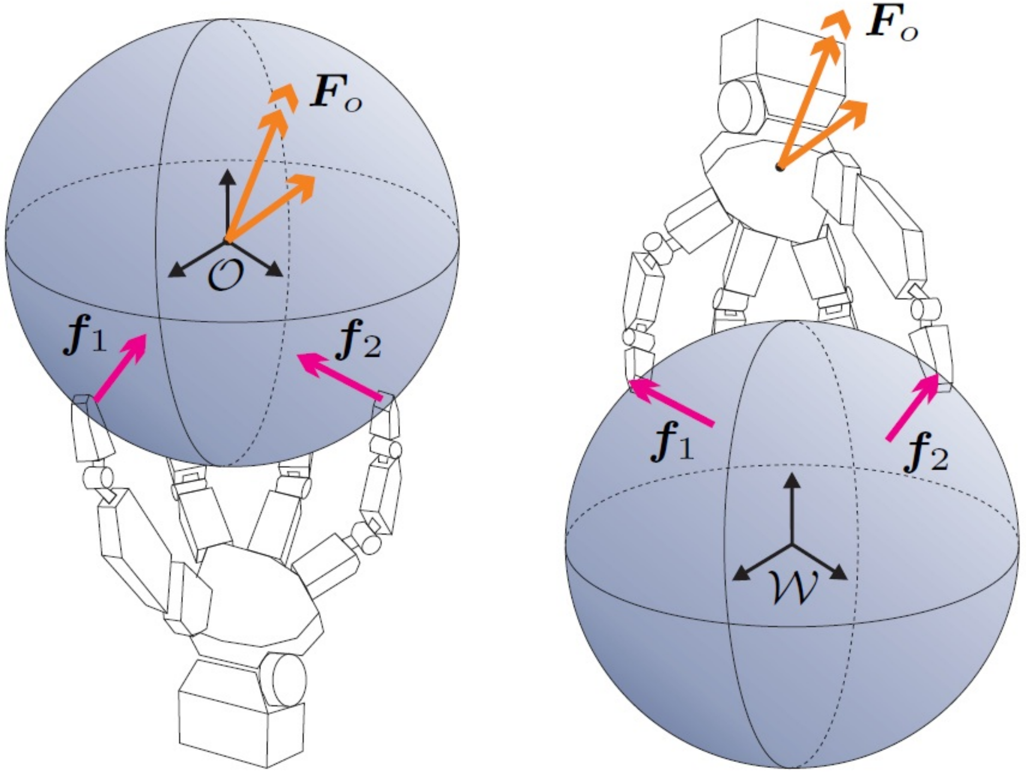


Grasping & Balancing

Force distribution: Similar problems!

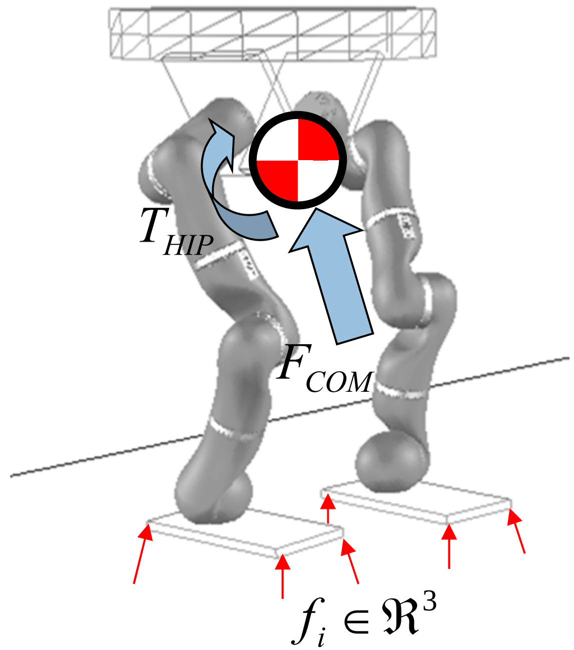


Force distribution

Relation between balancing wrench & contact forces

$$G_i = \begin{bmatrix} R_i \\ \hat{p}_i R_i \end{bmatrix}$$

$$\boxed{W_d = \underbrace{\left[G_1 \cdots G_\eta\right]}_{\begin{bmatrix} G_F \\ G_T \end{bmatrix}} \underbrace{\begin{pmatrix} f_1 \\ \vdots \\ f_\eta \end{pmatrix}}_{f_C}}$$



Constraints:

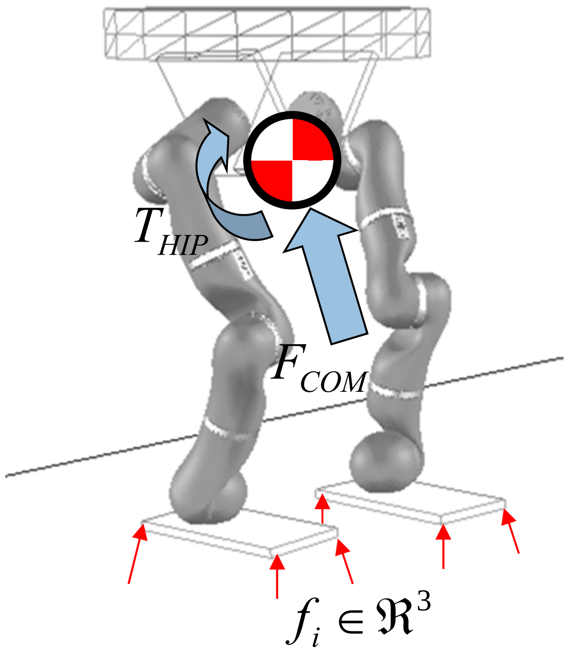
- Unilateral contact: $f_{i,z} > 0$ (implicit handling of ZMP constraints)
- Friction cone constraints

Force distribution

Relation between balancing wrench & contact forces

$$G_i = \begin{bmatrix} R_i \\ \hat{p}_i R_i \end{bmatrix}$$

$$\boxed{W_d = \underbrace{\left[G_1 \cdots G_\eta\right]}_{\begin{bmatrix} G_F \\ G_T \end{bmatrix}} \underbrace{\begin{pmatrix} f_1 \\ \vdots \\ f_\eta \end{pmatrix}}_{f_C}}$$



Constraints:

- Unilateral contact: $f_{i,z} > 0$ (implicit handling of ZMP constraints)
- Friction cone constraints

Formulation as a constraint optimization problem

$$f_C = \arg\min \left\{ \alpha_1 \left\| F_{COM} - G_F f_C \right\|^2 + \alpha_2 \left\| T_{HIP} - G_T f_C \right\|^2 + \alpha_3 \left\| f_C \right\|^2 \right\} \qquad \alpha_1 >> \alpha_2 >> \alpha_3$$

Contact force control via joint torques

Multibody robot model:

COM as a base coordinate → system structure with decoupled COM dynamics.

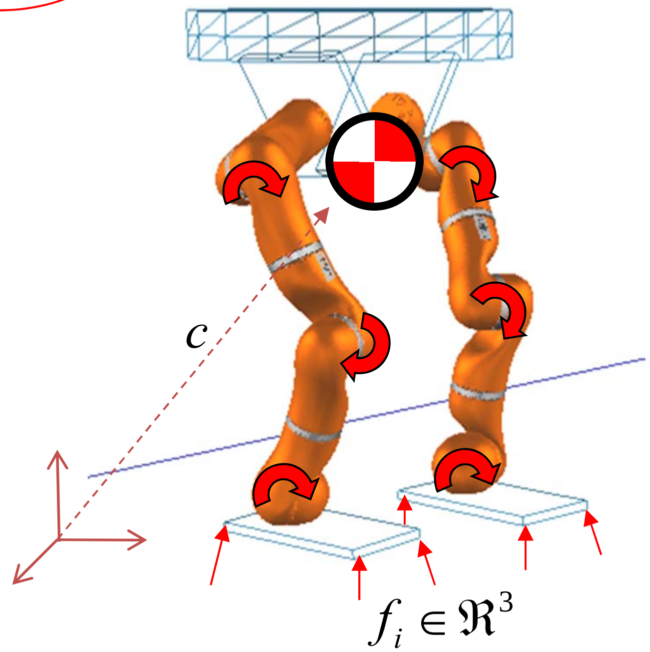
[Space Robotics], [Wieber 2005, Hyon et al. 2006]

$$\begin{bmatrix} M & 0 \\ 0 & \hat{M}(q) \end{bmatrix}\begin{pmatrix} \ddot{c} \\ \ddot{\hat{q}} \end{pmatrix} + \begin{bmatrix} 0 \\ \hat{C}(\hat{q},\dot{\hat{q}}) \end{bmatrix} + \begin{bmatrix} -Mg \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} - \sum_{i=r,l} \begin{bmatrix} I & 0 \\ J_i(\hat{q})^T & \end{bmatrix} F_i$$

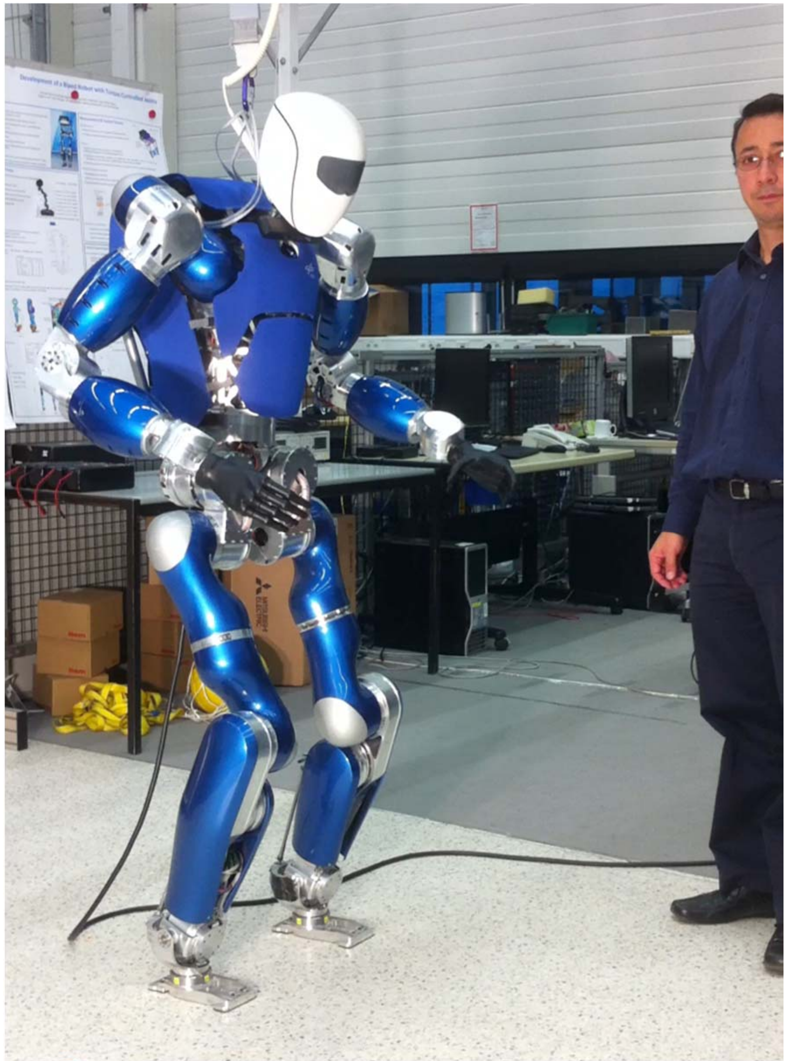
$$M\ddot{c} = Mg - \sum f_i$$

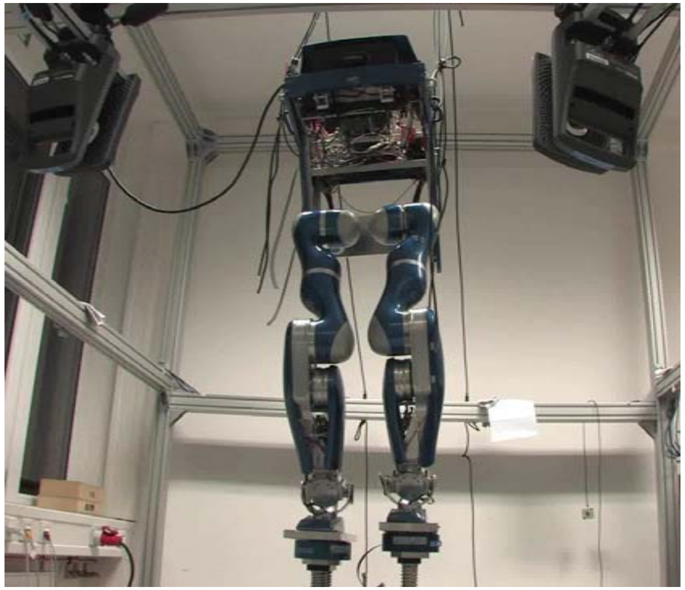
$$\boxed{\tau = \sum J_i(\hat{q})^T f_i}$$

Passivity based compliance control
(well suited for balancing)



Experiments





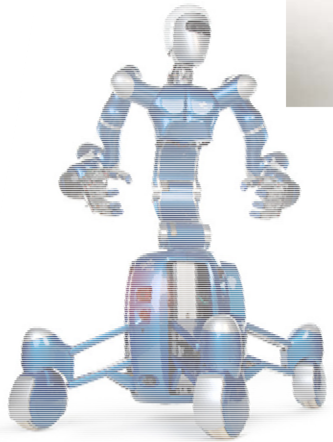
Extension to Multi-contact scenarios

Current work by Bernd Henze

Whole body control (no separation between upper body and lower body)

Summary

1) Compliance control framework for robots with joint torque sensors → implies robot model with joint flexibilty



2) Multi-objective compliance control:
→Hierarchical nullspace velocities to achieve
decoupling without feedback cancellation

3) Extension to legged robots & multi-contact interaction

Thank you very much for your kind attention!

DLR

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