The stack of tasks

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The stack of tasks

Outline

Introduction

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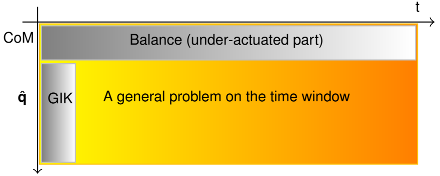
The stack of tasks provides a control framework for real-time redundant manipulator control

- implementation of a data-flow,
- control of the graph by python scripting,
- task-based hierarchical control,
- portable: tested on HRP-2, Nao, Romeo.

Motion generation: the general problem

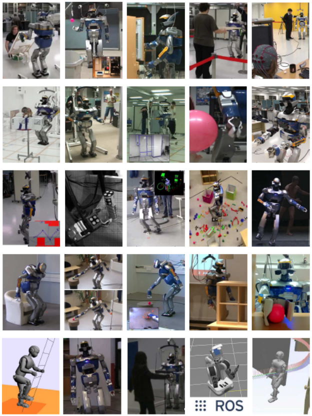
Non-linear problem

$$\begin{cases} \min f(\mathbf{q}(t), \mathbf{v}(t)) \\ \mathbf{g}(\mathbf{q}(t), \mathbf{v}(t)) < 0 \\ \mathbf{h}(\mathbf{q}(t), \mathbf{v}(t)) = 0 \end{cases}$$

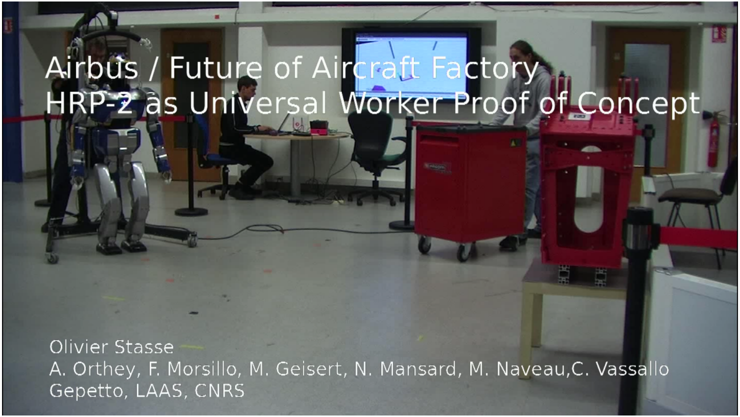


$$\begin{cases} \mathbf{M}_1(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}_1(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}_1(\mathbf{q}) = \mathbf{T}_1(\mathbf{q})\mathbf{u} + \mathbf{C}_1^\top(\mathbf{q})\lambda & \text{Actuated dynamics of the robot} \\ \mathbf{M}_2(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}_2(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}_2(\mathbf{q}) = \mathbf{C}_2^\top(\mathbf{q})\lambda & \text{Underactuated dynamics of the robot} \\ g(\ \lambda) \geq 0 & \text{General balance criteria} \\ \mathbf{u}_{min} < \mathbf{u} < \mathbf{u}_{max} & \text{Torques limits} \\ \hat{\mathbf{q}}_{min} < \hat{\mathbf{q}} < \hat{\mathbf{q}}_{max} & \text{Joints limits} \\ d(\mathcal{B}_i(\mathbf{q}), \mathcal{B}_j(\mathbf{q})) > \epsilon, \forall p(i,j) \in \mathcal{P} & \text{(self-)collisions} \end{cases}$$

Applications



Applications with several features



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Rigid body $\mathcal{B}$

- Configuration represented by an homogeneous matrix

$$M_{\mathcal{B}} = \begin{pmatrix} R_{\mathcal{B}} & \mathbf{t}_{\mathcal{B}} \\ 0\ 0\ 0 & 1 \end{pmatrix} \in SE(3)$$

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Point $\mathbf{x} \in \mathbb{R}^3$ in local frame of $\mathcal{B}$ is moved to $\mathbf{y} \in \mathbb{R}^3$ in global frame:

$$\begin{pmatrix} \mathbf{y} \\ 1 \end{pmatrix} = M_{\mathcal{B}} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix}$$

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and

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- Velocity of point P on $\mathcal{B}$

$$\mathbf{v}_p = \dot{\mathbf{t}}_{\mathcal{B}} + \omega_{\mathcal{B}} \times \vec{O_{\mathcal{B}}P}$$

where $O_{\mathcal{B}}$ is the origin of the local frame of $\mathcal{B}$.

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$$\mathbf{q}_{waist} = (x, y, z, roll, pitch, yaw)$$

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- Position of $\mathcal{B}_i$ depends on $\mathbf{q}$:

$$M_{\mathcal{B}_i}(\mathbf{q}) \in SE(3)$$

Velocity

- Velocity:

$$
\begin{aligned}
\dot{\mathbf{q}} &= (\dot{x}, \dot{y}, \dot{z}, \omega_x, \omega_y, \omega_z, \dot{\theta}_1, \cdots \dot{\theta}_{n-6}) \\
\omega &\in \mathbb{R}^3
\end{aligned}
$$

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- Velocity of $\mathcal{B}_i$

$$\begin{pmatrix} \mathbf{v}_{\mathcal{B}_i} \\ \omega_{\mathcal{B}_i} \end{pmatrix} (\mathbf{q}, \dot{\mathbf{q}}) = J_{\mathcal{B}_i}(\mathbf{q}).\dot{\mathbf{q}} \in \mathbb{R}^6$$

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- Definition: function of the
  - robot configuration,
  - time and
  - possibly external parameters

  that should converge to 0:

$$T \in C^{\infty}(\mathcal{C} \times \mathbb{R}, \mathbb{R}^m)$$

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$$T(\mathbf{q}, t) = \begin{pmatrix} \mathbf{t}(M^{*-1}(t)M(\mathbf{q})) \\ u_{\theta}(R^{*-1}(t)R(\mathbf{q})) \end{pmatrix}$$

  where
  - $\mathbf{t}()$ is the translation part of an homogeneous matrix,
  - R and R^{*} are the rotation part of M and M^{*}.

Hierarchical task based control

Given

- a configuration **q**,
- two tasks of decreasing priorities:
  - $T_1 \in C^\infty(\mathcal{C} \times \mathbb{R}, \mathbb{R}^{m_1})$,
  - $T_2 \in C^\infty(\mathcal{C} \times \mathbb{R}, \mathbb{R}^{m_2})$,

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compute a control vector $\dot{\mathbf{q}}$

- that makes T_1 converge toward 0 and
- that makes T_2 converge toward 0 if possible.

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We try to enforce

- $\dot{T}_1 = -\lambda_1 T_1 \quad \Rightarrow \quad T_1(t) = e^{-\lambda_1 t} T_1(0) \rightarrow 0$

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- λ_1 and λ_2 are called the gains associated to T_1 and T_2.

Moore Penrose pseudo-inverse

Given a matrix $A \in \mathbb{R}^{m \times n}$, the Moore Penrose pseudo inverse $A^+ \in \mathbb{R}^{n \times m}$ of A is the unique matrix satisfying:

$$\begin{aligned} AA^+A &= A \\ A^+AA^+ &= A^+ \\ (AA^+)^T &= AA^+ \\ (A^+A)^T &= A^+A \end{aligned}$$

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$$Ax = b, \quad A \in \mathbb{R}^{m \times n}, \ x \in \mathbb{R}^n, \ \ b \in \mathbb{R}^m$$

$x = A^+b$ minimizes

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- $\|Ax - b\|$ over $\mathbb{R}^n$,
- $\|x\|$ over $\mathrm{argmin}\|Ax - b\|$.

Hierarchical task based control

Resolution of the first constraint:

$$\dot{T}_1 = J_1 \dot{\mathbf{q}} \quad + \quad \frac{\partial T_1}{\partial t} = -\lambda_1 T_1 \tag{1}$$

$$J_1 \dot{\mathbf{q}} \quad = \quad -\lambda_1 T_1 - \frac{\partial T_1}{\partial t} \tag{2}$$

$$\dot{\mathbf{q}}_1 \quad \triangleq \quad -J_1^{+}(\lambda_1 T_1 + \frac{\partial T_1}{\partial t}) \tag{3}$$

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Where J_1^+ is the (Moore Penrose) pseudo-inverse of J_1.

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Resolution of the first constraint:

$$\dot{T}_1 = J_1 \dot{\mathbf{q}} + \frac{\partial T_1}{\partial t} = -\lambda_1 T_1 \quad (1)$$

$$J_1 \color{red}{\dot{\mathbf{q}}} = -\lambda_1 T_1 - \frac{\partial T_1}{\partial t} \quad (2)$$

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Where J_1^{+} is the (Moore Penrose) pseudo-inverse of J_1.
$\dot{\mathbf{q}}_1$ minimizes

- $\|J_1 \dot{\mathbf{q}} + \lambda_1 T_1 + \frac{\partial T_1}{\partial t}\| = \|\dot{T}_1 + \lambda_1 T_1\|$

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Hence,

- if $\lambda_1 T_1 + \frac{\partial T_1}{\partial t}$ is in $Im(J_1)$, (1) is satisfied

Hierarchical task based control

In fact

$$\forall u \in \mathbb{R}^n, \;\; J_1 \left(\dot{\mathbf{q}}_1 + (I_n - J_1^+ J_1) u\right) = J_1 \dot{\mathbf{q}}_1$$

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therefore,

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_1 + (I_n - J_1^{+} J_1) u$$

also minimizes $\| J_1 \dot{\mathbf{q}} + \lambda_1 T_1 + \frac{\partial T_1}{\partial t} \|$.

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$P_1 = (I_n - J_1^+ J_1)$ is a projector on J_1 kernel:
$J_1 P_1 = 0$

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$J_1 P_1 = 0$
$\forall u \in \mathbb{R}^n$, if $\dot{\mathbf{q}} = P_1 u$, then, $\dot{T}_1 = \frac{\partial T_1}{\partial t}$.

Controlling the second task

We have

$$
\begin{aligned}
\dot{\mathbf{q}} &= \dot{\mathbf{q}}_1 + P_1 u \\
\dot{T}_2 &= J_2 \dot{\mathbf{q}} + \frac{\partial T_2}{\partial t} \\
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\end{aligned}
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Thus

$$\begin{aligned}-\lambda_2 T_2 &= J_2\dot{\mathbf{q}}_1 + \frac{\partial T_2}{\partial t} + J_2 P_1 u \\ J_2 P_1 u &= -\lambda_2 T_2 - J_2\dot{\mathbf{q}}_1 - \frac{\partial T_2}{\partial t}\end{aligned}$$

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\end{aligned}
$$

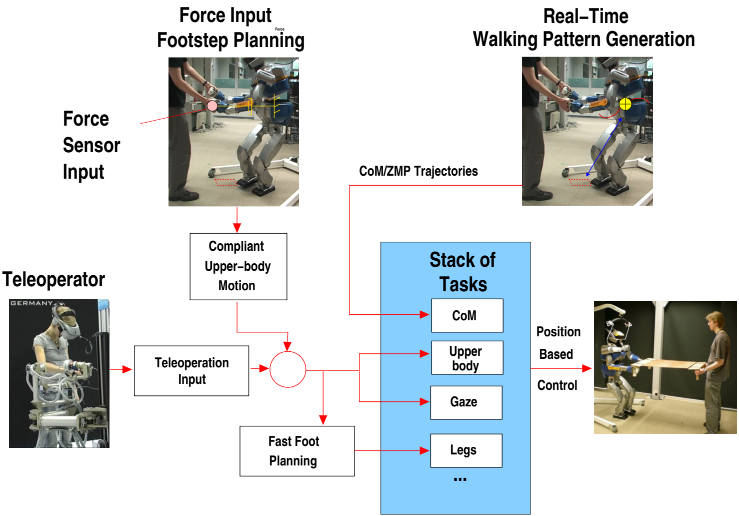
$$
\begin{aligned}
u &= -(J_2 P_1)^{+}(\lambda_2 T_2 + J_2 \dot{\mathbf{q}}_1 + \frac{\partial T_2}{\partial t}) \\
\dot{\mathbf{q}}_2 &\triangleq \dot{\mathbf{q}}_1 + P_1 u \\
&= \dot{\mathbf{q}}_1 - P_1 (J_2 P_1)^{+}(\lambda_2 T_2 + J_2 \dot{\mathbf{q}}_1 + \frac{\partial T_2}{\partial t}))
\end{aligned}
$$

minimizes $\|\dot{T}_2 + \lambda_2 T_2\|$ over $\dot{\mathbf{q}}_1 + Ker\ J_1$.

Advanced formulation

- Inverse Dynamics
- Weighted Pseudo-inverse
  - Faster (!?) computation
  - Easier to formulate
  - Do not guarantee convergence
  - Difficulty to tune the weights
  - Do not handle properly inequalities
- Hierarchical Quadratic Program
  - Slower (!?) computation time
  - Warranty on priority
  - Handle easily inequalities
  - Difficult to formulate (here hidden in the solver)
  - Known problems with cycles and singularities management

Example: Human-humanoid robot interaction

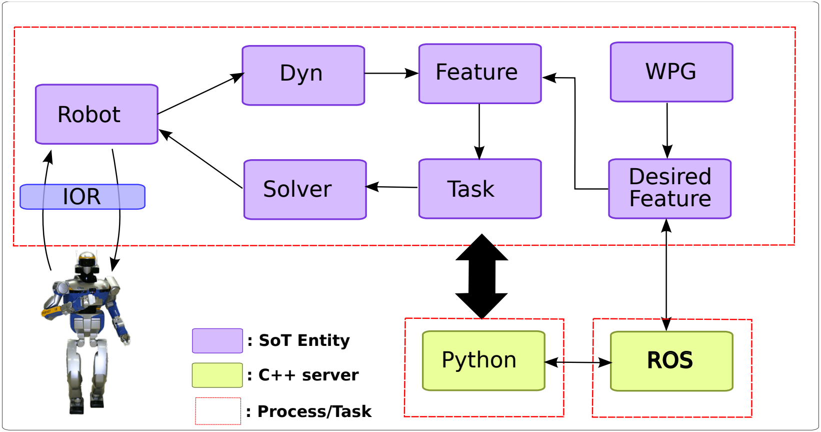


ROBOT@CWE

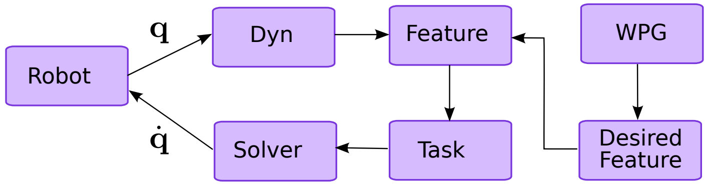

ROBOT CWE
MULTI-MODAL COLLABORATIVE WORK WITH HUMANOID ROBOTS: A CASE STUDY WITH HRP-2
O. STASSE, P. EVRARD, N. MANSARD, P. GERGONDET, A. KHEDDAR, K. YOKOI, E. YOSHIDA, T. SCHAUSS, C. PASSENBERG, A. PEER, M. BUSS, A. WEISS, M. TSCHELIGI, E. GRIBOVSKAYA, A. BILLARD, L. BLASI, J. GANCET, M. SEGARRA.
JRL, CNRS/AIST, LSR, TUM, PLUS, LASA, EPFL, HP ITALIANA, SAS, DRAGADOS

Outline

Software structure - Conceptual view



Software structure - Link with Model



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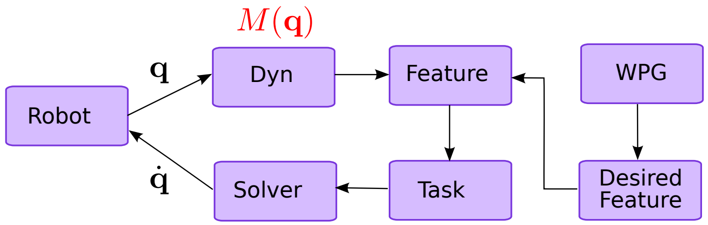

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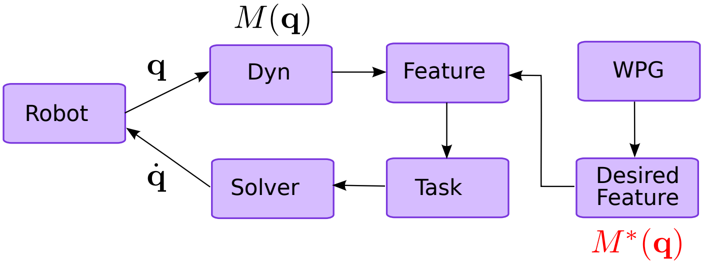

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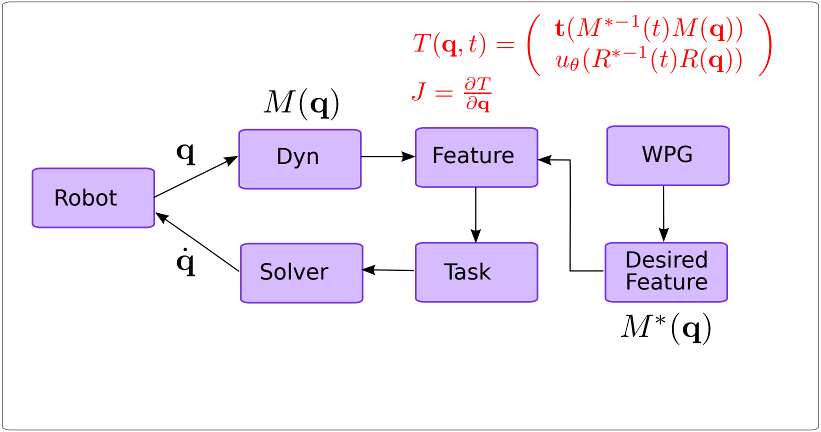

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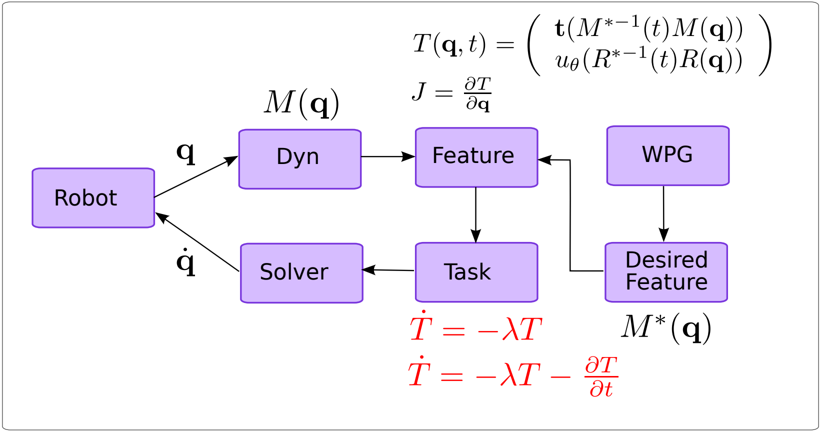

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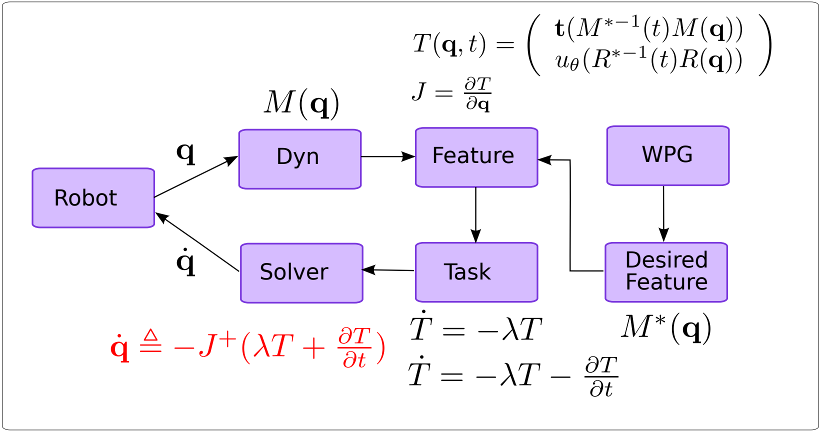
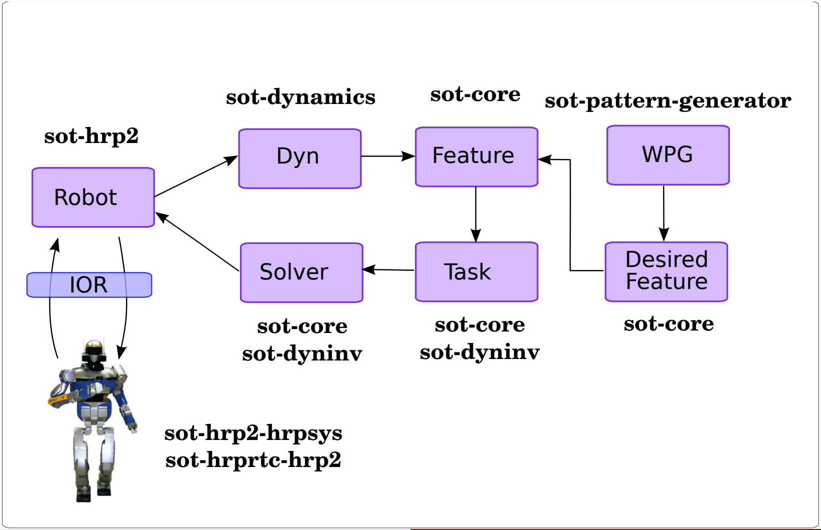
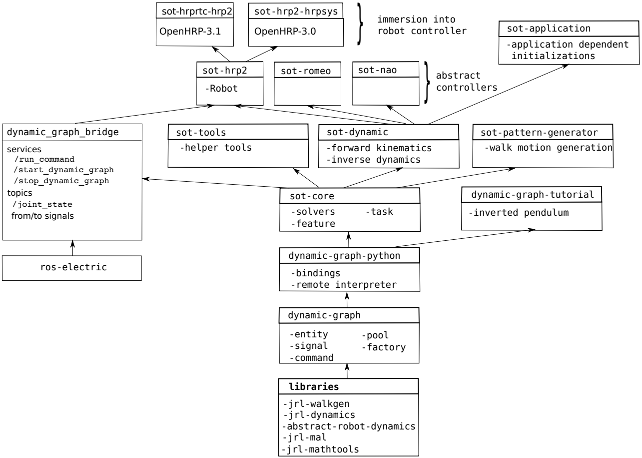


Software structure - Repositories



Architecture overview



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dynamic-graph

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- Pool
  - stores all instances of entities,
  - return reference to entity of given name.

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- reading an out-dated signal triggers recomputation.
- New types can be dynamically added (advanced)

Command

Asynchronous interface

- input in a fixed set of types,
- trigger an action,
- returns a result in the same set of types.

dynamic-graph-python

Python bindings to `dynamic-graph`

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  - class Signal
    - property `value` to set and get signal value
- remote interpreter to be embedded into a robot controller (advanced)

`dynamic-graph-tutorial`

Simple use case for illustration

- Definition of 2 entity types
  - `InvertedPendulum`
    - input signal: `force`
    - output signal: `state`
  - `FeedbackController`
    - input signal: `state`
    - output signal: `force`

dynamic-graph-tutorial

```
>>> from dynamic_graph.tutorial import InvertedPendulum, FeedbackController
>>>
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>>> a = InvertedPendulum ('IP')
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--- <IP> signal list:
|-- <Sig:InvertedPendulum(IP)::input(double)::force (Type Cst) AUTOPLUGGED
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Classical inverted pendulum dynamic model

List of commands:
-----------------
  getCartMass:         Get cart mass
  getPendulumLength:   Get pendulum length
  getPendulumMass:     Get pendulum mass
  incr:                Integrate dynamics for time step provided as input
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>>> a.help ('incr')
incr:

    Integrate dynamics for time step provided as input

      take one floating point number as input

>>>
```

dynamic-graph-tutorial

Package provides

- C++ code of classes `InvertedPendulum` and `FeedbackController`,

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- information about how to create a python module defining the bindings in cmake,
- python script that runs an example.

sot-core

Class `FeatureAbstract`

- function of the robot and environment states

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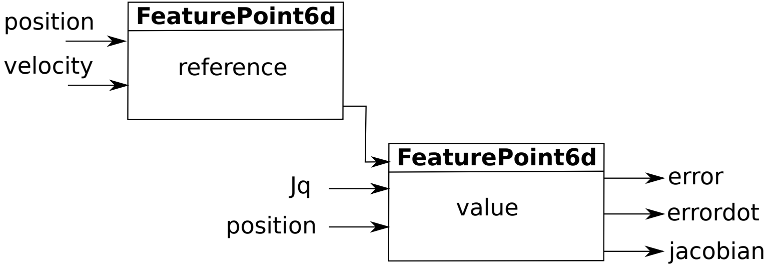
- with a mapping e from G into $\mathbb{R}^m$ such that

$$e(0_G) = 0$$

Feature

When paired with a reference, features become *tasks*.

- Example



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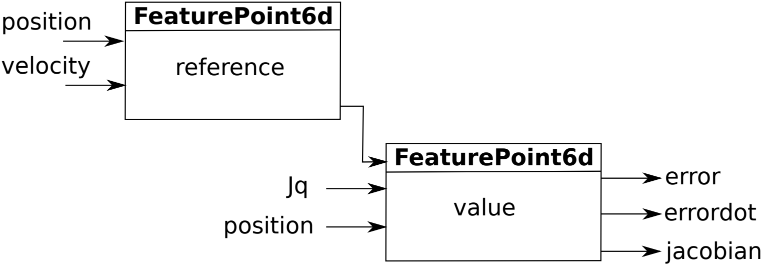

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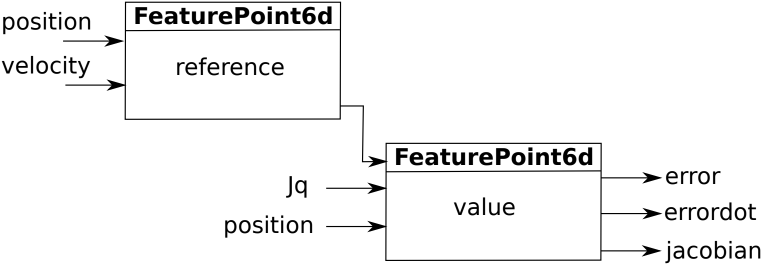
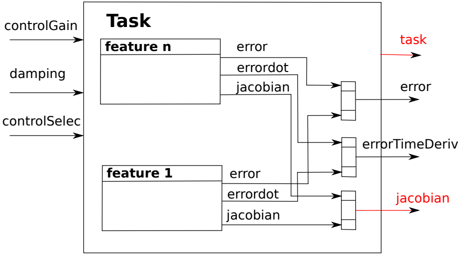


- $\texttt{error} = e\,(\texttt{value.position} \ominus \texttt{reference.position})$
- `errordot`: derivative of `error` when `value.position` is constant.

Task

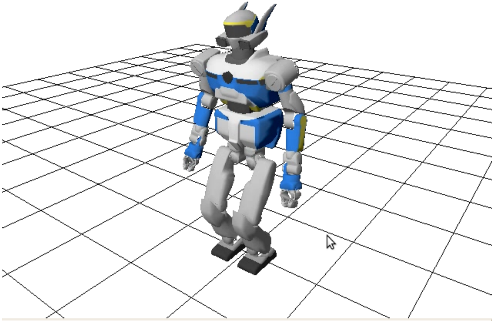
- Collection of features with a control gain,
- implements abstraction `TaskAbstract`



- $\mathtt{task} = -\mathtt{controlGain}.\mathtt{error}$

Solver SOT

Hierarchical task solver



- computes robot joint velocity

sot-dynamic

`dynamic_graph.sot.dynamics.Dynamic` builds a kinematic chain from a file and

- computes forward kinematics
  - position and Jacobian of end effectors (wrists, ankles),
  - position of center of mass
- computes dynamics
  - inertia matrix.

`sot-pattern-generator`

`dynamic_graph.sot.pattern_generator`

- Entity `PatternGenerator` produces walk motions as
  - position and velocity of the feet
  - position and velocity of the center of mass

sot-application

`dynamic_graph.sot.application`

- Provide scripts for standard control graph initialization
  - depends on application: control mode (velocity, acceleration)

Packages specific to robots

`sot-hrp2`

- defines a class `Robot` that provides
  - ready to use features for feet, hands, gaze and center of mass,
  - ready to use tasks for the same end effectors,
  - an entity `Dynamic`,
  - an entity `Device` (interface with the robot control system)

`sot-hrprtc-hrp2`

- provide an RTC component to integrate sot-hrp2 into the robot controller.

Utilities

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- `MetaTaskCom`:

Installation

Through `robotpkg`

```
git clone http://trac.laas.fr/git/robots/robotpkg.git
cd robotpkg
./bootstrap/bootstrap --prefix=<your_prefix>
cd motion/sot-dynamic

make install
```

Installation

Through github:

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git clone --recursive git://github.com/jrl-umi3218/jrl-mal.git
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  ```
- for each package,
  ```
  mkdir package/build
  cd package/build
  cmake -DCMAKE_INSTALL_PREFIX=<your_prefix> ..

  make install
  ```
  ```

Installation

Through installation script

```
git clone git://github.com/stack-of-tasks/install-sot.git
cd install-sot/scripts

./install_sot.sh
```

Conclusion

- Pro
  - Generic to put instantaneous controller together
  - Allow code reusability,
  - Real-time performance
  - Adapted to complex applications
- Cons
  - The current project management needs improvment
  - Better binary packages support (in progress)
  - Eigen support (but no performance improvment to be expected)

Perspectives

- Whole body model predictive control
- Multi-core architecture

Thank you for your attention !

We are welcoming questions, constructive feedback and help for the Stack Of Tasks.

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